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Gains from interest-rate smoothing in a small open economy with zero-bound aversion

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ABSTRACT

We extend the Monacelli [Monacelli, T. (2005). Monetary policy in a low pass-through environment. *Journal of Money, Credit and Banking*, *37*(6), 1047–1066] model to allow for a central bank that penalizes nominal interest rate paths that are too close to the zero lower bound. We analytically derive the optimal interest-rate policy rule in each equilibrium under four policy regimes: (i) benchmark commitment to an ex-ante optimal monetary-policy plan; (ii) benchmark discretionary policy; (iii) optimal delegation to a discretionary policy maker with similar preferences to society; and (iv) optimal delegation to a discretionary policy maker with an additional taste for interest-rate smoothing. Under the commitment benchmark, the optimal interest-rate rule is proved to be intrinsically inertial, whereas this property is non-existent under discretionary policy. In the absence of commitment, there are gains to delegating policy to an interest-rate smoothing central banker. We show that while the endogenous law of one price gap in the model exacerbates the optimal policy trade-off that arises under discretionary policy, the latter feature of interest-rate smoothing acts to weaken it, by mimicking intrinsic inertia under the commitment policy.

1. Introduction

Empirically, the level of nominal interest rates for many industrialized small open economies tend to be highly and positively autocorrelated. For example, Espinosa-Vega and Rebucci (2003) document first-order autocorrelations for nominal interest rates in these countries that are near random walk. Furthermore, these rates are perfectly correlated with the respective countries' monetary policy rates.

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Often this feature is rationalized as central banks' preference for interest-rate smoothing. It may also be the case that such persistence in policy rates arise naturally out of monetary policy that is *ex-ante* optimal, without any explicit desire for smoothing policy (see Woodford, 1999, 2003b). The former hypothesis then raises the question of whether, and when, such explicit preference for interest rate smoothing has any gain for society in a small open economy.

In a closed economy model without any endogenous monetary policy trade off, Woodford (1999) showed there are gains to society for an explicit interest-rate smoothing objective to be incorporated into the central bank's objective when it cannot commit to an ex-ante optimal monetary policy plan. This is because the interest rate smoothing component of the objective in the latter regime induces an optimal policy that approximates the commitment policy more closely. While Rogoff (1985) considered the delegation of monetary policy to a conservative central banker as a solution to the well-known average inflation bias problem, Woodford (1999) advocated hiring an interest-rate-smoothing central bank delegate as a solution to the stabilization bias problem which arises from lack of commitment by the policy maker. This latter result will be conveniently labeled as the *Woodford proposition* below.

In this paper, we extend the Monacelli (2005) model to consider the case where the monetary policy maker penalizes domestic nominal interest rate paths that are too close (from above) to zero. This not unrealistic assumption, as in Woodford (2003b, see Chapter 4.2), is used as a reduced-form way of bounding the stochastic paths of the interest rate above zero. This has an interpretation of aversion to the zero-interest-rate lower bound by policy makers.[1] While we could explicitly model occasionally binding zero-lower-bound constraints (see e.g. Adam & Billi, 2006, 2007) that is not the purpose here in this paper. Instead, we apply the approach of Woodford (2003b) which allows us to analytically derive the optimal interest-rate policy rule in the equilibria for four policy regimes.

To that end, we consider notions of optimal policy under the following policy regimes: (i) benchmark commitment to an ex-ante optimal monetary-policy plan; (ii) discretion, or ex-post lack of commitment to (i); and a Rogoff-style delegation of discretionary policy to an independent policy maker that either (iii) shares the same family of loss functions as society, or (iv) has an additional interest-rate smoothing term in its objective function. Under the commitment benchmark, the optimal interest-rate rule is proved to be intrinsically inertial, whereas this property is non-existent under discretionary policy. Analytical and numerical results show that under discretion, there are gains to delegating policy to an interest-rate smoothing central banker. Specifically, we show that in this Monacelli (2005) economy, endogenous deviations from the law of one price exacerbates the optimal CPI inflation and output-gap trade-off in discretionary monetary policy. However, we also show that by delegating policy to an explicit interest-rate smoothing policy maker, this trade-off can be weakened. This weakening of the trade-off can be interpreted as a forced encoding of history dependence in the policy decision that approximates policy under commitment. Our result is robust to alternative degrees of exchange rate pass through, the types of shocks impinging the natural rate, and minor departures from optimal pricing behavior.

One might expect the important insight on the value of interest-rate smoothing of Woodford (1999) to carry through to small-open-economy monetary theory and policy. This would indeed be true in the case of typical small-open-economy models with complete exchange-rate pass through in the style of Clarida, Galí, and Gertler, (2001) and Galí and Monacelli (2002). Clarida et al. (2001) showed that the small-open-economy optimal monetary policy rule and resulting equilibrium is qualitatively the same as its closed economy counterpart.[2] However, such a conclusion may not be warranted in many small open economies that experience incomplete exchange rate pass through, and this question has not been theoretically analyzed for such economies. In fact, Monacelli (2005) shows that because of incomplete pass through, monetary policy via the interest rate path also affects the paths of CPI inflation (i.e. both domestic and imported goods price inflation) and output gap via the channel of the exchange rate and the deviation from the law of one price. Monacelli (2005) showed that it is no

[1] We thank an anonymous referee for refining this point.

[2] Intuitively, with perfect exchange-rate pass through, any volatility in the exchange rate gets transmitted to aggregate demand immediately via the terms of trade and is thus captured in the output-gap stabilization objective of the central bank. Meanwhile, nominal rigidity in domestic goods prices can be dealt with by domestic-goods inflation targeting.

longer optimal to just achieve flexible domestic producer prices (which would have been sufficient in the closed-economy case), but there is a trade off between stabilizing domestic producer prices on the one hand, and stabilizing either the output gap or the law-of-one-price gap on the other.

We thus address the unanswered question of whether the *Woodford proposition* survives in a more general case of a small open economy with incomplete exchange-rate pass through where the optimal monetary policy is clearly one that cannot merely stabilize domestic goods inflation. This question is important since a large number of advanced industrialized countries, for instance Canada, Australia, the United Kingdom, and New Zealand, are small open economies that face exactly this kind of problem where exchange rate pass through is less than complete.

The remainder of the paper is organized as follows. We describe the New Keynesian (NK) model of Monacelli (2005) briefly in Section 2. We then motivate why, in the small open economy that we employ, optimal interest-rate smoothing or not can be crucial, in Section 2.2. Optimal monetary policy under commitment is considered and contrasted with alternative optimal discretionary policies in Section 3. In Section 3, we also prove and derive the intuition behind how the *Woodford proposition* helps to weaken the effect of the endogenous law-of-one-price gap on the equilibrium policy trade-off. We provide numerical examples showing that the *Woodford proposition* carries through to this economy as well in Section 4. We conclude in Section 5.

2. The Monacelli NK model

Denote the aggregate exogenous state vector as some $\mathbf{z}_t$. Let $\mathbf{X}_t(\mathbf{z}_t):=(\pi_t, \pi_{H,t}, \pi_{F,t}, \tilde{y}_t, \psi_{F,t}, r_t)$ denote the state-contingent vector containing CPI inflation, home goods inflation, foreign goods inflation, home output gap, law of one price gap, and home nominal interest rate, respectively. The NK small-open-economy model of Monacelli (2005) can be described by the following set of log-linearized approximate equilibrium conditions. We present the microfoundations of this linearized system in Appendix A. For each state $\mathbf{z}_t$ and date $t \in \mathbb{N}:=\{0, 1, 2, \ldots\}$,

$$\pi_t = (1-\gamma)\pi_{H,t} + \gamma\pi_{F,t} \tag{1}$$

$$\pi_{H,t} = \beta\mathbb{E}_t\pi_{H,t+1} + \kappa_y\tilde{y}_t + \kappa_\psi\psi_{F,t} \tag{2}$$

$$\pi_{F,t} = \beta\mathbb{E}_t\pi_{F,t+1} + \lambda_F\psi_{F,t} \tag{3}$$

$$\tilde{y}_t = \mathbb{E}_t\tilde{y}_{t+1} - \frac{\omega_s}{\sigma}(r_t - \mathbb{E}_t\pi_{H,t+1} - r_t^n) + \Gamma_y\mathbb{E}_t(\psi_{F,t+1} - \psi_{F,t}) \tag{4}$$

$$\psi_{F,t} = \mathbb{E}_t\psi_{F,t+1} - r_t + r_t^* + \mathbb{E}_t\pi_{F,t+1} + \mathbb{E}_t\pi_{t+1}^*. \tag{5}$$

The first identity (1) is the consumer price index inflation derived from a CES aggregator of domestic and imported goods prices. Domestic (2) and imported goods (3) inflation are, respectively, described by NK Phillips curves derived from the Calvo (1983) staggered price setting model. The dynamic IS curve (4) is a result of household intertemporal optimization and market clearing. Lastly, the dynamics of deviations from the law of one price (or LOP gap) is given by (5).[3]

Note that the other variables (r_t^n, r_t^*, π_t^*), respectively, the domestic natural rate of interest, world nominal interest rate, and world CPI inflation, can be shown to be exogenous in the sense that there are affine functions of $\mathbf{z}_t$, or a component of it.

[3] We can also construct the evolution of the nominal exchange rate using the uncovered interest-parity condition,

$$\mathbb{E}_t e_{t+1} = e_t + r_t - r_t^*, \tag{6}$$

and the terms of trade and the real exchange rate will be, respectively, defined by

$$s_t = \frac{\sigma}{\omega_s}\tilde{y}_t + \left[\frac{\sigma(1+\varphi)}{\sigma+\varphi\omega_s}\right](z_t - z_t^*) + \frac{\sigma}{\omega_s}\left[\frac{\omega_\psi - \omega_s}{\sigma+\varphi\omega_s} - \omega_s\right]\psi_{F,t}, \tag{7}$$

and

$$q_t = \psi_{F,t} + (1-\gamma)s_t. \tag{8}$$

The “deep” parameters are $(\sigma, \varphi, \eta, \gamma, \theta_H, \theta_F)$, respectively denoting the constant-relative-risk-aversion coefficient for the utility of consumption, the inverse of the real-wage elasticity of labor supply, the elasticity of substitution between domestic and foreign goods, the foreign goods share in CPI, and the one period probabilities that domestic-goods firms and imports retailers do not change their prices in their Calvo (1983) optimal staggered-pricing model. The coefficients of this system are nonlinear functions of underlying taste and technology parameters. Specifically, $\lambda_H = \theta_H^{-1}(1-\theta_H)(1-\beta\theta_H)$, $\lambda_F = \theta_F^{-1}(1-\theta_F)(1-\beta\theta_F)$, $\kappa_y = \lambda_H(\varphi + (\sigma/\omega_s)) > 0$, $\kappa_\psi = \lambda_H(1-(\omega_\psi/\omega_s)) \geq 0$, $\Gamma_y = ((\gamma(1-\gamma)(\sigma\eta-1))/\sigma) > 0$ and $\omega_s = 1+\gamma(2-\gamma)(\sigma\eta-1) \geq \omega_\psi = 1+\gamma(\sigma\eta-1) > 0$.

There are two exogenous stochastic processes $\mathbf{z}_t := (z_t^*, z_t)$ – technology shock in the rest of the world and its counterpart in the small open economy – given by first-order Markov processes:

$$\mathbf{z}_t = \mathbf{M}\mathbf{z}_{t-1} + \boldsymbol{\upsilon}_t, \quad \boldsymbol{\upsilon}_t \sim i.i.d.(\mathbf{0}, \boldsymbol{\Sigma}), \quad \mathbf{M} := \begin{pmatrix} \rho^* & 0 \\ 0 & \rho \end{pmatrix}, \tag{9}$$

where $\mathbf{M}$ is a stable matrix. The natural rate of interest depends on relative productivity shocks in the small open economy and the rest of the world:

$$r_t^n = -\frac{\sigma(1+\varphi)(1-\rho)}{\sigma+\varphi\omega_s}\left[\left(\frac{(\omega_s-1)\varphi}{\sigma+\varphi}\right) z_t^* + z_t\right]. \tag{10}$$

Definition 1. A rational expectations equilibrium in the small open economy is a set of bounded stochastic processes $\{\pi_t, \pi_{H,t}, \tilde{y}_t, \pi_{F,t}, \psi_{F,t}, r_t\}_{t\in\mathbb{N}}$ that satisfies the system of Eqs. (1)–(5) for any given set of processes $\{r_t^n, r_t^*, \pi_t^*, z_t, z_t^*\}_{t\in\mathbb{N}}$.

2.1. Relation to the canonical NK model

The well-known Clarida et al. (2001) and Galí and Monacelli (2002) model with perfect exchange rate pass through to domestic prices is nested within this model. Specifically, when $\theta_F = 0$ and $\psi_{F,t} = 0$ for all $t \in \mathbb{N}$ and all $\mathbf{z}_t$, the law of one price holds for imported goods, the relevant system becomes

$$\pi_{H,t} = \beta\mathbb{E}_t\pi_{H,t+1} + \kappa_y\tilde{y}_t, \tag{11}$$

$$\tilde{y}_t = \mathbb{E}_t\tilde{y}_{t+1} - \frac{\omega_s}{\sigma}(r_t - \mathbb{E}_t\pi_{H,t+1} - r_t^n), \tag{12}$$

and (10), which is qualitatively similar, and often termed isomorphic, to the canonical NK closed-economy model (see e.g. Clarida, Galí, & Gertler, 1999; Woodford, 2003a). What is important to note here is that in this simpler model, any policy setting r_t faces no inflation-output-gap trade-offs in terms of stabilizing shocks to the natural rate, r_t^n. In fact, in the absence of any exogenous shock to the Phillips curve (11), there are no policy trade-offs at all.

In the Monacelli (2005) model, there is an imperfect exchange rate pass through channel (indexed by θ_F) which allows shocks to the natural rate, r_t^n to induce an endogenous “shock” to the Phillips curve. This is the key to the endogenous policy trade-off in the model. We elaborate on this further in the next section. Further, in Section 3, we will also show that this additional channel exacerbates the optimal discretionary monetary policy trade-off in the Monacelli (2005) model; and how delegation to an interest smoothing central banker can weaken this trade-off.

2.2. Policy objective trade-offs in the Monacelli NK model

In the complete pass-through canonical NK model given by (11)–(12), shocks to output gap via the natural rate, (10), can be completely offset by the nominal interest rate without the repercussion on domestic producer prices. Thus the role of optimal interest rate smoothing, either as a result of commitment or discretion, merely acts as a speed brake on the straightforward adjustment process, just as in the closed economy model of Woodford (1999). This is clearly not the case in the Monacelli (2005) model.

As shown by Monacelli (2005), the existence of an incomplete exchange-rate pass through to retail imports prices creates an endogenous cost-push shock, in the form of the LOP gap, $\psi_{F,t}$, to the aggregate supply relations in (2) and (3). Consider a one-unit domestic technology shock in (9). By inspecting the dynamic IS relation in (4) and (10), we can see that the output gap ought to decrease by the amount $k = (\sigma + \varphi\omega_s)^{-1} \times [(1+\varphi)(1-\rho)\omega_s + \omega_s(1+\varphi)]$, if all else including future expectations are held constant. The fall in output gap would cause both domestic and imported goods inflation to fall via the real marginal cost channel in (2) and (3), since output is demand determined. However, one may think such real technology shocks can be neutralized by cutting the interest rate, by the amount $\sigma\omega_s^{-1} \times k$ so that output gap in (4) remains unchanged. This would be true if there were no gaps in the law of one price. However, when $\theta_F > 0$, a fall in the interest rate by the said amount, would cause a positive LOP gap via (5). This will further feed through to decrease output gap in (4) since the composite parameter $\kappa_\psi > 0$, when $\eta > 1$. Furthermore, the positive LOP gap will cause both domestic- and foreign-goods inflation measures to rise. Therefore, conditional on the parameterization of the model, there is a trade-off in the usual inflation-output space implied by the LOP gap which creates an endogenous cost-push shock under what would otherwise be efficient technology shocks. In summary, by attempting to neutralize shocks to output gap and thus domestic goods inflation, the central bank ends up trading off output gap with domestic and foreign goods inflation, or, as Monacelli (2005) showed, CPI inflation.

Thus, whether a central bank smooths the interest rate or not under optimal policy matters even more in the Monacelli (2005) kind of economy. On the one hand, if policy is not smoothed, one may expect more abrupt and larger trade-offs in terms of inflation volatility versus output gap or LOP gap volatility. On the other hand, if monetary policy is smoothed, one may obtain the opposite. We will show that relative to the benchmark commitment case, the outcome favors optimal interest-rate smoothing because it helps to reduce the trade-offs under discretionary policy, in Section 4.

3. Optimal monetary policy with zero bound aversion

In this section, we begin by characterizing what is meant as "optimal policy". First, as the benchmark we consider the optimal commitment policy (COM). Against this, we will also consider time-consistent optimal policies (in a Markov perfect equilibrium sense), as in Woodford's proposition.[4] Second, we characterize the outcome if the central bank cannot commit to the policy in COM. We denote this as discretion (DIS). Third, we consider the outcome if the central bank cannot commit anyway, society may optimally delegate discretionary policy to another central bank with the same family of preferences. We call this optimal delegation (DEL1). Fourth, we characterize an alternative optimal delegation regime (DEL2), whereby the policy delegate has a different policy preference – it also explicitly prefers to stabilize changes in the interest rate in addition to the social preference. We will show analytically how this feature alters the Markov perfect equilibrium monetary-policy trade off in the model.

The social *ex-ante* loss function is

$$W(\mathbf{z}_0) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t L_t, \tag{13}$$

where $\beta \in (0, 1)$. Following Monacelli (2005), we let CPI inflation and output gap enter as arguments in the per-period loss function, but with an additional term penalizing fluctuations in the level of the nominal interest rate:

$$L_t = \pi_t^2 + b_w \tilde{y}_t^2 + b_r(r_t - r^*)^2, \tag{14}$$

[4] It should be noted that even when the output level in the long run is made efficient, stabilization bias (a short-run business cycle phenomenon) can still exist when the central bank cannot optimally commit to a once-and-for-all policy. To abstract from the problem of an average inflation bias it is assumed, as shown in Galí and Monacelli (2002), that fiscal policy in the long run provides a subsidy to real wage of $\tau = (1/\varepsilon)$ where ε is the common elasticity of substitution between differentiated goods. This yields output at steady state which equals the first-best equilibrium outcome so that the target output gap is zero. Having done this, the focus can then be solely on the welfare effects of the stabilization bias problem.

where $r^* > 0$.[5] The weights $b_w > 0$ and $b_r > 0$, respectively, denote the relative concern of the central bank for output gap, as in the specification of Monacelli (2005), and, interest rate variability around $r^* > 0$. The latter is our interpretation of society's and the central bank's aversion to interests rates that approach zero from above. One may envision that these weights have been chosen by society and given to the central bank at some initial "constitutional design" stage.

3.1. Regime 1: Commitment

First we consider the policy regime where it is assumed that the central bank can commit to an ex-ante optimal plan. The commitment regime (denoted as COM) is defined by the problem where the central bank can commit to a plan $\{\mathbf{X}_t(\mathbf{z}_t)\}_{t\in\mathbb{N}}$ that minimizes (13) and (14) subject to the constraints of the evolution of the economy in (1)–(5). The first-order conditions for the central bank's problem are then:

$$(1-\gamma)\pi_t + \phi_{1,t} - \phi_{1,t-1} - \frac{\omega_s}{\beta\sigma}\phi_{2,t-1} = 0 \tag{17}$$

$$b_w\tilde{y}_t - \kappa_y\phi_{1,t} + \phi_{2,t} - \beta^{-1}\phi_{2,t-1} = 0 \tag{18}$$

$$b_r(r_t - r^*) + \frac{\omega_s}{\sigma}\phi_{2,t} - \phi_{4,t} = 0 \tag{19}$$

$$-\kappa_\psi\phi_{1,t} + \Gamma_y(\phi_{2,t} - \beta^{-1}\phi_{2,t-1}) - \lambda_F\phi_{3,t} + \beta^{-1}\phi_{4,t-1} - \phi_{4,t} = 0 \tag{20}$$

$$\gamma\pi_t + \phi_{3,t} - \phi_{3,t-1} + \beta^{-1}\phi_{4,t-1} = 0 \tag{21}$$

with the initial conditions $\phi_{1,-1} = \phi_{2,-1} = \phi_{3,-1} = \phi_{4,-1} = 0$.

The existence of lagged Lagrange multipliers in (17)–(21) implies that endogenous variables and in particular the optimal interest-rate instrument under commitment must not only react to current shocks, but also past movements of endogenous variables. This creates intrinsic policy inertia, independent of the serial correlation of exogenous stochastic processes as Woodford (1999) had shown in the case of a typical closed-economy New Keynesian model. This is stated in Proposition 1 below.

Proposition 1. *The competitive equilibrium with rational expectations (RE) under regime COM is induced by an optimal interest rate rule which is backward and forward looking in terms of current and past RE forecasts of domestic and foreign technology shocks:*

$$r_t = (1-\rho_r)r^* + \rho_r r_{t-1} - \mathbf{\Theta}_b\sum_{s=0}^{t-1}\mathbf{N}^s\mathbf{C}\sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F}\mathbf{M}^j\mathbf{z}_{t-s-1} - \mathbf{\Theta}_f\sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F}\mathbf{M}^j\mathbf{z}_t, \tag{22}$$

where $\mathbf{\Theta}_b$, $\mathbf{\Theta}_f$, $\mathbf{N}$, $\mathbf{C}$, $\mathbf{H}$, and $\mathbf{F}$ are matrices obtain under the RE equilibrium. It is also intrinsically inertial and the inertia coefficient ρ_r is independent of the structure of serial correlation in $\mathbf{z}_t := (z_t^*, z_t)$.

Proof. See Appendix B. □

[5] As Woodford (2003a) showed, if welfare measures are defined up to the second order, then the zero-interest-rate lower bound is sufficiently characterized by the following constraints

$$\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t r_t\right\} \geq 0, \tag{15}$$

$$\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t r_t^2\right\} \leq \left(1+\frac{1}{k^2}\right)\left[\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t r_t\right\}\right]^2. \tag{16}$$

These two constraints say that the "average" value of the sequence $\{r_t\}_{t\in\mathbb{N}}$ must be nonnegative, and its fluctuations are bounded by k standard deviations above zero. Woodford (2003a) shows that the problem of minimizing (13) subject to (15), (16) and the structural Eqs. (1)–(5) can be replaced by one where the period loss function is given by (14).

It can also be seen in Appendix B that $\boldsymbol{\Theta}_b$ and $\boldsymbol{\Theta}_f$ are decreasing in absolute terms with the central banker's preference for interest rate stability, b_r, in the case of pre-commitment. In other words, when the central bank places greater weight on interest-rate variability, it reacts less aggressively to current and expected future domestic and foreign productivity shocks.

3.2. Regime 2: Discretion

Second, consider the regime when the central bank cannot commit to the once and for all optimal plan (denoted as DIS). The regime is given by the problem where, in each period, the central bank minimizes (14) subject to the constraints (1)–(5). In this case, the central bank under discretion has an incentive to disregard the lagged constraints in (17)–(21), and this is consistent with the beliefs of the private sector in a Markov perfect equilibrium. The resulting first-order conditions now are:

$$(1-\gamma)\pi_t + \phi_{1,t} = 0 \tag{23}$$

$$b_w\tilde{y}_t - \kappa_y\phi_{1,t} + \phi_{2,t} = 0 \tag{24}$$

$$b_r(r_t - r^*) + \frac{\omega_s}{\sigma}\phi_{2,t} - \phi_{4,t} = 0 \tag{25}$$

$$-\kappa_\psi\phi_{1,t} + \Gamma_y\phi_{2,t} - \lambda_F\phi_{3,t} - \phi_{4,t} = 0 \tag{26}$$

$$\gamma\pi_t + \phi_{3,t} = 0 \tag{27}$$

Definition 2. A Markov perfect equilibrium is a stochastic process $\{\pi_t, \pi_{H,t}, \tilde{y}_t, \pi_{F,t}, \psi_{F,t}, r_t, e_t\}_{t\in\mathbb{N}}$ that satisfies (1)–(6) and (23)–(27) for all $t\in\mathbb{N}$, for any given set of exogenous stochastic processes $\{r_t^n, r_t^*, \pi_{t+1}^*, z_t, z_t^*\}_{t\in\mathbb{N}}$.

Under the case of optimal discretion, one can find an analytical expression for the interest-rate rule in the model. This turns out to be a Taylor-type rule where the rule reacts to both CPI inflation and output gap. However, the elasticity of policy with respect to these arguments are constrained by the private and policy-preference parameters, reflecting the credibility constraint under a Markov perfect equilibrium. This is summarized by the following proposition.

Proposition 2. *The Markov perfect equilibrium that solves the problem in* Definition 2 *for regime DIS yields an optimal Taylor-type rule:*

$$r_t = r^* + \Phi_\pi\pi_t + \Phi_y\tilde{y}_t \tag{28}$$

where $\Phi_\pi = b_r^{-1}\{(1-\gamma)[\kappa_\psi + \kappa_y\sigma^{-1}(1+\gamma(\sigma\eta-1)) + \gamma\lambda_F]\}$ *and* $\Phi_y = b_w b_r^{-1}\sigma^{-1}[1+\gamma(\sigma\eta-1)]$ *are positive.*

There is no intrinsic inertia in the optimal interest-rate policy (28) since no lagged Lagrange multiplier terms appear in the first-order condition for the central bank's optimal choice. This is simply because the central banker in each period has no incentive to be bound by the constraints from past periods.

Suppose, without loss of generality, at any time $t\in\mathbb{N}$, $r_t = r^*$. Then the Markov perfect equilibrium policy rule (28) encodes monetary-policy trade-offs between CPI inflation and output gap, given by

$$\pi_t = -\left(\frac{\Phi_y}{\Phi_\pi}\right)\tilde{y}_t := -\tau\tilde{y}_t, \tag{29}$$

which suggests the usual trade-off between inflation and output-gap stabilization. However, unlike in the closed-economy models (or its isomorphic small open economy versions), the trade-off τ now is also dependent on the endogenous law-of-one-price gap, reflected in the composite parameter $\lambda_F > 0$. In particular, the greater is the degree of imperfect exchange rate pass through, θ_F, and hence λ_F, the more output gap variation must be tolerated for smaller variations to CPI inflation. However, note that the monetary-policy trade off $\tau > 0$ is independent of the policy maker's aversion to the zero bound, b_r. This will provide a clue for comparison with the regime DEL2 (in Section 3.4) where society delegates discretionary policy to a policy maker that has explicit taste for interest rate smoothing. It is this latter feature that will soften the policy trade off arising from the policy maker's temptation to deviate along with the rational expectations enforcement of the Markov perfect equilibrium.

3.3. Regime 3: Optimal delegation

Third, consider the regime DEL1 when society delegates policy to a central banker who cannot commit to the once and for all optimal plan. The optimal delegation regime is defined by a choice of loss function weights (b_w^*, b_r^*) such that for each fixed set of weights, the delegate central bank indexed by (b_w^*, b_r^*) minimizes (14) subject to the constraints (1)–(5); and the weights minimize the social loss (13) and (14) as defined for Regime 1 (COM). The Markov perfect equilibrium in this regime has similar characterization as that in Regime 2 (DIS), except for the fact that the particular solution will be indexed by the optimal weights (b_w^*, b_r^*).

Proposition 3. *The Markov perfect equilibrium for the regime DEL1 yields an optimal Taylor-type rule:*

$$r_t = r^* + \Phi_\pi^* \pi_t + \Phi_y^* \tilde{y}_t \tag{30}$$

where $\Phi_\pi^* = (b_r^*)^{-1}\{(1-\gamma)[\kappa_\psi + \kappa_y \sigma^{-1}(1+\gamma(\sigma\eta - 1)) + \gamma\lambda_F]\}$ and $\Phi_y = b_w^*(b_r^*)^{-1}\sigma^{-1}[1+\gamma(\sigma\eta-1)]$ *are positive.*

Again, the policy trade-off now faced by an optimally delegate policy maker, will be similar to that discussed in Section 3.2. However, society can pick a central banker with a different preference indexed by (b_w^*, b_r^*) so that the trade-off can be minimal in terms of society's loss.

3.4. Regime 4: Optimal delegation and interest-rate smoothing

Fourth, consider society delegating policy to a central banker with an additional preference for interest rate smoothing. Denote this regime as DEL2.

Specifically, assume that each central banker indexed by policy preference weights $(b_w, b_{\Delta r})$, under discretion, is one who minimizes

$$L_t^{CBsmooth} = \pi_t^2 + b_w \tilde{y}_t^2 + b_{\Delta r}(r_t - r_{t-1})^2 \tag{31}$$

subject to the constraints of private variables in (1)–(5). Notice that rather than having an objective with an interest-rate target or variability term, $b_{\Delta r} > 0$ indexes a concern for changes in the interest rate. Since this is also given in quadratic form, it means that the larger the changes in interest rate between two periods, the more the central bank is penalized in terms of its loss per period, $L_t^{CBsmooth}$. We omit displaying the case of regime DIS since the outcomes would be dominated in the social loss sense by the regime DEL1.

Similar to Regime 3 (DEL1), society now selects some optimal weights $(b_w^*, b_{\Delta r}^*)$ such that the particular delegate indexed by $(b_w^*, b_{\Delta r}^*)$ will induce a Markov perfect equilibrium that yields the lowest possible social loss.

The first-order conditions for a delegate central banker are the same as (23)–(27), except that (25) is replaced with:

$$b_{\Delta r}(r_t - r_{t-1}) + \frac{\omega_s}{\sigma}\phi_{2,t} - \phi_{4,t} = 0 \tag{32}$$

Proposition 4. *The Markov perfect equilibrium in regime DEL2 induces an optimal difference Taylor-type rule of the form*

$$r_t = r_{t-1} + \Phi_\pi^* \pi_t + \Phi_y^* \tilde{y}_t \tag{33}$$

where

$$\Phi_\pi^* = (b_{\Delta r}^*)^{-1}\{(1-\gamma)[\kappa_\psi + \kappa_y \sigma^{-1}(1+\gamma(\sigma\eta-1)) + \gamma\lambda_F]\} > 0$$
$$\Phi_y^* = b_w^*(b_{\Delta r}^*)^{-1}\sigma^{-1}[1+\gamma(\sigma\eta-1)] > 0.$$

Proof. This is a straightforward result from amending Proposition 2 for interest-rate growth in the first-order conditions; specifically in (32). □

However, notice that now an additional pre-determined state variable r_{t-1} enters the credibility constrained optimal rule. This is simply an artefact of the central banker's explicit interest rate smoothing objective which constrains the optimal time-consistent policy. Further, the concern for interest-rate changes $b_{\Delta r}$ affects the optimal policy responses Φ_π^* and Φ_y^* such that a greater concern for smoother interest rates entails less response to CPI inflation and output gap, *ceteris paribus*.

Although $b_{\Delta r}$ does not alter the trade-off faced by the delegate central bank, the existence of a preference for interest-rate smoothing now alters the trade-off as follows. Similar to the discussion in Section 3.2, consider fixing the current triple $(r_t, \pi_t, \tilde{y}_t)$ as induced by the Markov perfect equilibrium in the DEL2 regime. The augmented optimal trade-off faced by the delegate is this case, compared to (29), is

$$\pi_t = -\left(\frac{\Phi_y^*}{\Phi_\pi^*}\right)\tilde{y}_t + \frac{1}{\Phi_\pi^*}(r_t - r_{t-1}). \tag{34}$$

This second term on the right-hand-side of (34) thus acts as a vertical shift to the $(\tilde{y}_t, \pi_t)$ trade-off locus. Intuitively, (34) says that for any outcome of a current Markov perfect equilibrium policy, say $r_t = r^*$, the current policy trade-off inherits completely the policy decision and outcome from the previous period, so that there needs to be only a smaller change in inflation in return for a given sacrifice of output gap, since $r_{t-1} > 0$. In other words, the delegation of discretionary policy to the explicit interest-rate smoothing central banker, in regime DEL2, results in a softening of the current policy trade-off, compared to that in regime DIS or DEL1. In other words, the optimal policy in regime DEL2 approximates (imperfectly) the policy under the regime COM in the sense of history dependence. Moreover, the greater is the concern for interest-rate smoothing $b_{\Delta r}$, the greater is the impact (via Θ_π^{*-1}) of the r_{t-1} term in softening the inflation-output-gap trade-off in (34), for any equilibrium current choice of r_t.

Proposition 5. *For every given current Markov perfect equilibrium outcome in Regime DEL2 indexed by* $(r_t, \pi_t, \tilde{y}_t)$, *the current optimal monetary policy trade-off* (34) *involves a smaller sacrifice in CPI inflation for a given output gap variation compared to Regimes DIS and DEL1, and, this gain is further improved by a greater concern for interest-rate smoothing* $b_{\Delta r}$.

We will compute the effect of this result, in the following section, using numerical examples.

4. Simulation results

In this section we numerically evaluate the outcomes under discretionary policy (DIS, DEL1 and DEL2) against the benchmark commitment outcome (COM), whose theoretical properties were presented in Section 3. The main message from these numerical examples is that the Woodford proposition (as summarized in Proposition 5) can be extended to the Monacelli (2005) small open economy with endogenous policy trade-offs.

Specifically, the social loss and business cycle effect of delegating monetary policy (DEL2) to a central banker with an explicit taste for interest-rate smoothing (31) is considered. This is considered alongside equilibrium outcomes for the regimes COM, DIS and DEL1. The three discretionary-policy regimes (DIS, DEL1 and DEL2) are evaluated using the same social loss function (13)–(14) as in COM. Then, some robustness examples for optimal policies are considered for different degrees of exchange-rate pass through, alternative assumptions of exogenous shocks and the existence of some backward-looking inflation dynamics. Lastly, we show using impulse response analysis how the regime DEL2 best approximates the regime COM.

4.1. Statistic for policy regime comparisons

Following Jensen (2002), our welfare statistic will be measured as the square root of the difference in society's loss function value under a given discretionary policy, W_D, and society's loss function value under the assumption that a central bank can commit, once and for all, to minimizing society's true

Table 1
Policy regimes, delegation and welfare: baseline economy.

	Regimes:			
	COM $b_w = 0.5$ $b_r = 0.2$	DIS $b_w = 0.5$ $b_r = 0.2$	DEL1 $b_w^* = 0.25$ $b_r^* = 0.1$	DEL2 $b_w^* = 0.41$ $b_{\Delta r}^* = 0.28$
Outcomes				
Social loss ($\times 100$)	0.105	1.166	0.486	0.280
Pre-commitment gain, $\tilde{\pi}$	0	1.030	0.617	0.418
Standard deviation:				
Interest rate, r_t	0.045	0.148	0.119	0.079
Interest rate change, Δr_t	0.029	0.117	0.093	0.052
Output gap, $\tilde{y}_t$	0.022	0.022	0.022	0.031
Nominal exchange rate, e_t	1.787	2.409	2.221	1.894
CPI inflation, π_t	0.020	0.084	0.043	0.033
Real exchange rate, q_t	0.897	0.803	0.819	0.814

Note: (a) Policy weights in DEL1 and DEL2 are optimal delegation weights.

social loss, W_{COM}:

$$\tilde{\pi}_D = \sqrt{W_D - W_{\text{COM}}}, \qquad D \in \{DIS, DEL1, DEL2\} \tag{35}$$

This has the interpretation of the gain of moving from a discretionary policy regime D to one with full commitment to society's loss valuation, in terms of the amount of compensation in inflation required to achieve some target inflation rate.

4.2. Parameterization

Parameter values are set out as follows. The private sector parameters are the same as Monacelli (2005). The common rate of time preference is set as $\beta = 0.99$. The coefficient of relative risk aversion is set as $\sigma = 1$, implying a log period utility in consumption. The elasticity of substitution between home and foreign goods is given by $\eta = 1.5$. Labor supply elasticity is given by $\varphi = 3$ while price stickiness in both domestic and retail imports sectors are assumed equal, and they take on the standard value of $\theta_H = \theta_F = 0.75$. This implies average price-stickiness of 4 quarters. The degree of openness in the economy, governed by the imports share in the consumption basket is given by $\gamma = 0.4$. There are only two exogenous stochastic processes given by technology shocks domestically and abroad. The persistence parameter for both processes are $\rho = \rho^* = 0.9$ and their standard deviations are assumed to be one, $\sigma_z = \sigma_z^* = 1$. Finally society's loss function (14) is parameterized as $b_w = 0.5$, following Monacelli (2005), and we also set a lower value of $b_r = 0.2$. The loss function parameters are not inconsistent with those used in the applied literature.

4.3. Results

Table 1 summarizes the effect on our measure of social loss and the volatility (standard deviation) of the variables in the model under the different policy regimes, given the benchmark parameterization of the model economy. The variables are the nominal one-period interest rate, r_t, the change in this interest rate, Δr_t, output gap, $\tilde{y}_t$, nominal exchange rate, e_t, CPI inflation, π_t, and the real exchange rate, q_t. Where applicable, the last two rows of the table refer to society's loss function value and the measure of stabilization bias, respectively. Column "COM" of Table 1 shows the equilibrium outcome under the pre-commitment case. This will be used as a yardstick against which columns "DIS", "DEL1" and "DEL2" will measure. These, respectively, are the regimes where the central bank acts in discretion under society's preferences, where society optimally delegates to a central banker who acts in discretion but shares the same functional form of society in (14), and where society optimally delegates to a central banker who acts in discretion but has an interest-rate smoothing objective (31). In the last two

Table 2
Targeting regimes, delegation and welfare under high pass through, $\theta_F = 0.4$.

	Regimes:			
	COM $b_w = 0.5$ $b_r = 0.2$	DIS $b_w = 0.5$ $b_r = 0.2$	DEL1 $b_w^* = 0.3$ $b_r^* = 0.1$	DEL2 $b_w^* = 0.44$ $b_{\Delta r}^* = 0.2$
Outcomes				
Social loss ($\times 100$)	0.139	0.419	0.269	0.195
Pre-commitment gain, $\tilde{\pi}$	0	0.529	0.361	0.237
Standard deviation:				
Interest rate, r_t	0.067	0.117	0.104	0.084
Interest rate change, Δr_t	0.045	0.092	0.081	0.060
Output gap, $\tilde{y}_t$	0.017	0.018	0.019	0.022
Nominal exchange rate, e_t	1.890	2.198	2.105	1.955
CPI inflation, π_t	0.019	0.036	0.019	0.018
Real exchange rate, q_t	0.880	0.879	0.880	0.879

Note: (a) Policy weights in DEL1 and DEL2 are optimal delegation weights.

columns, “DEL1” and “DEL2”, any asterisked central bank preference parameters denote an optimally chosen set of policy preferences under policy delegation.[6]

In Table 1, column “DIS”, it can be seen that when the central bank is charged with minimizing the intertemporal social loss, the equilibrium under discretionary policy generates a greater social loss than the case with commitment policy (column COM). Recall that the implied form of the interest-rate rule in this case is (28). Furthermore, the equilibrium under discretion yields greater volatility in most of the variables in the short run.

Now consider optimally delegating the task of discretionary policy to a central banker that places different weights on the target variables in the same loss function as society’s. Column “DEL1” shows that society’s loss is greatly reduced but is still about four times the loss under pre-commitment, or that the gain from moving from discretion to pre-commitment is less, but it cannot outperform the latter. It is also interesting to note that the optimal weights on output gap and interest rate are less than those of society’s. This can have the interpretation of a Rogoff (1985) conservative, but in the sense of a short-run stabilization bias.

Finally, when one considers an interest-rate smoothing regime for discretionary policy, social loss is reduced and the volatilities of the key variables are also much less than pure discretion under “DIS” and “DEL1”. That is, the gain from having pre-commitment is further weakened, when one considers optimally delegating the discretionary policy making to an interest-rate smoothing central banker (Column “DEL2”). This is the case where the implied interest-rate rule is a difference Taylor-type rule of the form in (33), which we showed in Section 3.4, had the property that it induces history dependence in policy decisions such that it weakens equilibrium monetary policy trade-off. In summary, discretionary flexible inflation targeting by an optimally delegated interest-rate smoothing central banker, can reduce the stabilization bias arising out of discretionary monetary policy.

4.4. *Some robustness experiments*

In the remainder sections, we repeat the previous numerical exercise taking into account different configurations of model parameters or structure.

4.4.1. *Degree of exchange-rate pass through to imports prices*

Tables 2 and 3 retain the benchmark parameter values of the model, except for the degree of exchange-rate pass through, θ_F. Here, we consider if the previous conclusion about the regimes, espe-

[6] We conduct a set of grid searches over a subset of the parameter space for either (b_w, b_r) or $(b_w, b_{\Delta r})$ given by $[0, 1] \times [0, 1]$ with 2500 points. The second-moment statistics are computed in the frequency domain using codes provided by Harald Uhlig and are described in Uhlig (1999). In each case, the model is solved using linearized perturbation methods as described in Uhlig (1999).

Table 3
Targeting regimes, delegation and welfare under low pass through, $\theta_F = 0.99$.

	Regimes:			
	COM $b_w = 0.5$ $b_r = 0.2$	DIS $b_w = 0.5$ $b_r = 0.2$	DEL1 $b^*_w = 0.9$ $b^*_r = 0.1$	DEL2 $b^*_w = 0.26$ $b^*_{\Delta r} = 0.1$
Outcomes				
Social loss ($\times 100$)	0.174	0.728	0.339	0.241
Pre-commitment gain, $\tilde{\pi}$	0	0.744	0.406	0.259
Standard deviation:				
Interest rate, r_t	0.069	0.153	0.105	0.087
Interest rate change, Δr_t	0.049	0.120	0.082	0.061
Output gap, $\tilde{y}_t$	0.024	0.041	0.012	0.026
Nominal exchange rate, e_t	1.637	2.456	2.075	1.875
CPI inflation, π_t	0.022	0.042	0.034	0.024
Real exchange rate, q_t	1.273	1.387	2.075	1.348

Note: (a) Policy weights in DEL1 and DEL2 are optimal delegation weights.

cially the one under discretion with interest-rate smoothing, holds for high and low exchange-pass through. In these cases, we consider new sets of optimized weights for the delegation cases – columns DEL1 and DEL2 – when θ_F changes. In Table 2, we have the case of high pass through, $\theta_F = 0.4$.[7] Table 3 shows the case of a low degree of pass through, $\theta_F = 0.99$. Comparison of these results can be made with the benchmark case in Table 1.

It can be seen that the same general pattern arises, regardless of the degree of exchange-rate pass through. Pure discretion in monetary policy under society's loss function is the worse off. However, delegation of policy, under discretion, to a central banker who cares about smaller interest-rate changes outperforms the other discretionary policy regimes considered. Specifically, the distance, in terms of social loss or the gain from commitment, between an equilibrium under a central banker with a smoothing objective, and one under pre-commitment is reduced.

4.4.2. *Individual exogenous shocks*

A second set of robustness experiments is presented in Tables 4–6 . Taking the parameterization of central bank preferences as given from Tables 1–3, we now consider "shutting off" one exogenous shock at a time. Case 1 in Tables 1–3 refer to the case when only domestic productivity shocks exist, so that $\sigma_z = 1$ and $\sigma_z^* = 0$. Case 2 in the same tables assume the scenario when only productivity shocks from the rest of the world matter, or $\sigma_z = 0$ and $\sigma_z^* = 1$. Again, the same general conclusions in favor of discretionary interest-rate smoothing arise in these cases.

4.4.3. *Existence of rule-of-thumb pricing*

Finally we consider the robustness of our previous conclusion to the existence of backward-looking inflation dynamics. Retaining the parameterization of the policy weights from the purely forward-looking benchmark model in Table 1, we consider allowing for hybrid New Keynesian Phillips curves in both the domestic and imported goods sectors. These essentially have a lagged term for inflation as well as a forward-looking term. These equations are:

$$\pi_{H,t} = \omega_0^H \mathbb{E}_t\{\pi_{H,t+1}\} + \omega_1^H \pi_{H,t-1} + \omega_2^H mc_{H,t}, \tag{36}$$

$$\pi_{F,t} = \omega_0^F \mathbb{E}_t\{\pi_{F,t+1}\} + \omega_1^F \pi_{F,t-1} + \omega_2^F \psi_{F,t}, \tag{37}$$

where $\omega_0^j = \beta\theta_j/\{\theta_j + (1-\lambda_r^j)[1-\theta_j(1-\beta)]\}$, $\omega_1^j = (1-\lambda_r^j)/\{\theta_j + (1-\lambda_r^j)[1-\theta_j(1-\beta)]\}$, and $\omega_2^j = [\lambda_r^j(1-\theta_j)(1-\beta\theta_j)]/\{\theta_j + (1-\lambda_r^j)[1-\theta_j(1-\beta)]\}$, for $j = H, F$. Appendix C explains how these Phillips

[7] A stable and unique rational expectations equilibrium could not be attained for $\theta_F \in (0, 0.4)$, for all policy regimes, given the other parameters of the model.

Table 4
Targeting regimes under benchmark pass through, $\theta_F = 0.75$, with alternative shocks.

		Regimes:			
		COM $b_w = 0.5$ $b_r = 0.2$	DIS $b_w = 0.5$ $b_r = 0.2$	DEL1 $b_w^* = 0.25$ $b_r^* = 0.1$	DEL2 $b_w^* = 0.41$ $b_{\Delta r}^* = 0.28$
Outcomes					
Social loss ($\times$100):	Case 1	0.062	1.019	0.356	0.154
	Case 2	0.043	0.148	0.130	0.125
Pre-commitment gain, $\tilde{\pi}$:	Case 1	0	0.970	0.542	0.303
	Case 2	0	0.324	0.295	0.286
Standard deviation:					
Interest rate, r_t:	Case 1	0.038	0.127	0.094	0.066
	Case 2	0.025	0.076	0.073	0.043
Interest rate change, Δr_t:	Case 1	0.027	0.100	0.074	0.044
	Case 2	0.011	0.060	0.057	0.029
Output gap, $\tilde{y}_t$:	Case 1	0.019	0.005	0.007	0.019
	Case 2	0.010	0.022	0.020	0.024
Nominal exchange rate, e_t:	Case 1	0.431	1.273	0.935	0.720
	Case 2	1.735	2.046	2.012	1.752
CPI inflation, π_t:	Case 1	0.013	0.083	0.042	0.022
	Case 2	0.016	0.008	0.004	0.024
Real exchange rate, q_t:	Case 1	0.552	0.537	0.564	0.570
	Case 2	0.707	0.597	0.594	0.581

Note: (a) Case 1 corresponds to $\sigma_z^* = 0$ and $\sigma_z = 1$ and Case 2 is when $\sigma_z^* = 1$ and $\sigma_z = 0$. (b) Optimized parameters $b_w^*, b_r^*, b_{\Delta r}^*$ are w.r.t. benchmark case with all shocks in Table 1.

curves are obtained. Notice that when the fraction of rule-of-thumb price setters is zero or $\lambda_r^j = 1$, (36) and (37) collapse to the benchmark model given by (50) and (53), respectively. Also, $\partial\omega_1^j/\partial\lambda_r^j = -\theta_j/\{\theta_j + (1-\lambda_r^j)[1-\theta_j(1-\beta)]\}^2 < 0$ implying that as the proportion of rule-of-thumb firms increase, the weight on lagged inflation, ω_1, will increase. Furthermore, the sum of the coefficients on the lead

Table 5
Targeting regimes under low pass through, $\theta_F = 0.4$, with alternative shocks.

		Regimes:			
		COM $b_w = 0.5$ $b_r = 0.2$	DIS $b_w = 0.5$ $b_r = 0.2$	DEL1 $b_w^* = 0.3$ $b_r^* = 0.1$	DEL2 $b_w^* = 0.44$ $b_{\Delta r}^* = 0.2$
Outcomes					
Social loss ($\times$100):	Case 1	0.071	0.300	0.166	0.114
	Case 2	0.069	0.120	0.103	0.081
Pre-commitment gain, $\tilde{\pi}$:	Case 1	0	0.479	0.308	0.207
	Case 2	0	0.226	0.184	0.110
Standard deviation:					
Interest rate, r_t:	Case 1	0.050	0.093	0.079	0.066
	Case 2	0.045	0.071	0.067	0.052
Interest rate change, Δr_t:	Case 1	0.035	0.073	0.062	0.047
	Case 2	0.028	0.056	0.053	0.037
Output gap, $\tilde{y}_t$:	Case 1	0.016	0.004	0.010	0.012
	Case 2	0.006	0.018	0.017	0.018
Nominal exchange rate, e_t:	Case 1	0.504	0.934	0.790	0.697
	Case 2	1.822	1.990	1.951	1.826
CPI inflation, π_t:	Case 1	0.009	0.035	0.019	0.014
	Case 2	0.016	0.006	0.001	0.010
Real exchange rate, q_t:	Case 1	0.616	0.617	0.620	0.617
	Case 2	0.628	0.625	0.624	0.626

Note: (a) Case 1 corresponds to $\sigma_z^* = 0$ and $\sigma_z = 1$ and Case 2 is when $\sigma_z^* = 1$ and $\sigma_z = 0$. (b) Optimized parameters $b_w^*, b_r^*, b_{\Delta r}^*$ are w.r.t. benchmark case with all shocks in Table 2.

Table 6
Targeting regimes under low pass through, $\theta_F = 0.99$, with alternative shocks.

		Regimes:			
		COM $b_w = 0.5$ $b_r = 0.2$	DIS $b_w = 0.5$ $b_r = 0.2$	DEL1 $b_w^* = 0.9$ $b_r^* = 0.1$	DEL2 $b_w^* = 0.26$ $b_{\Delta r}^* = 0.1$
Outcomes					
Social loss ($\times 100$):	Case 1	0.157	0.433	0.183	0.179
	Case 2	0.017	0.295	0.157	0.062
Pre-commitment gain, $\tilde{\pi}$:	Case 1	0	0.525	0.161	0.148
	Case 2	0	0.527	0.374	0.212
Standard deviation:					
Interest rate, r_t:	Case 1	0.067	0.131	0.086	0.079
	Case 2	0.013	0.080	0.061	0.037
Interest rate change, Δr_t:	Case 1	0.047	0.103	0.067	0.055
	Case 2	0.011	0.062	0.048	0.026
Output gap, $\tilde{y}_t$:	Case 1	0.021	0.039	0.012	0.024
	Case 2	0.012	0.011	0.001	0.011
Nominal exchange rate, e_t:	Case 1	0.715	1.310	0.855	0.833
	Case 2	1.473	2.077	1.891	1.679
CPI inflation, π_t:	Case 1	0.021	0.011	0.017	0.016
	Case 2	0.008	0.040	0.029	0.017
Real exchange rate, q_t:	Case 1	0.816	1.121	0.953	0.941
	Case 2	0.977	0.817	0.886	0.966

Note: (a) Case 1 corresponds to $\sigma_z^* = 0$ and $\sigma_z = 1$ and Case 2 is when $\sigma_z^* = 1$ and $\sigma_z = 0$. (b) Optimized parameters b_w^*, b_r^*, $b_{\Delta r}^*$ are w.r.t. benchmark case with all shocks in Table 3.

and lag inflation terms, $\omega_0^j + \omega_1^j \in [\beta, 1]$. Given that usually, $\beta \simeq 1$, this suggests approximately vertical long-run Phillips curves.

Table 7 applies the policy regimes and weights (optimally selected weights in the case of delegation) from the benchmark case in Table 1 to the model with some fraction of rule-of-thumb price makers in the domestic and imported goods sectors. In this example, the fraction of forward-looking firms are set to $\lambda_r^j = 0.8$, for $j = H, F$, implying that each period with a probability of 0.2, each firm is backward-looking. In attempting to study cases where $\lambda_r^j < 0.8$ for $j = H, F$, we find that there are no numerically stable and unique rational expectations equilibria across all targeting regimes such that we can make uniform comparisons. Therefore we set $\lambda_r^H = \lambda_r^F = 0.8$ and interpret this as a small deviation from full forward-looking behavior in the benchmark model. The same conclusion is obtained in the case of Table 7. Again, having some intrinsic inertia in the inflation processes does not seem to alter the

Table 7
Rule-of-thumb pricing as deviation from benchmark, $\lambda_r^H = \lambda_r^F = 0.8$.

	Regimes:			
	COM $b_w = 0.5$ $b_r = 0.2$	DIS $b_w = 0.5$ $b_r = 0.2$	DEL1 $b_w^* = 0.25$ $b_r^* = 0.1$	DEL2 $b_w^* = 0.41$ $b_{\Delta r}^* = 0.28$
Outcomes				
Social loss ($\times 100$)	0.115	2.272	0.744	0.288
Pre-commitment gain, $\tilde{\pi}$	0	1.469	0.793	0.416
Standard deviation:				
Interest rate, r_t	0.046	0.190	0.136	0.079
Interest rate change, Δr_t	0.033	0.144	0.104	0.053
Output gap, $\tilde{y}_t$	0.021	0.032	0.021	0.030
Nominal exchange rate, e_t	1.771	2.766	2.356	1.856
CPI inflation, π_t	0.022	0.122	0.060	0.034
Real exchange rate, q_t	0.906	0.800	0.822	0.820

Note: (a) Policy weights are from Table 1.

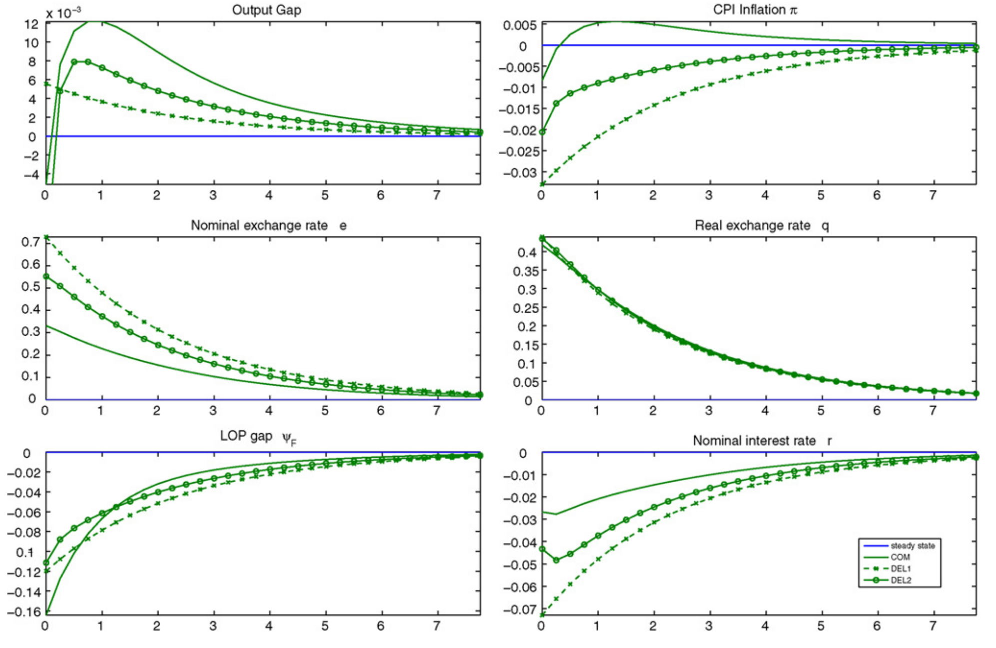

Fig. 1. Impulse responses to domestic technology shock: benchmark economy.

benefit of delegating discretionary monetary policy to an interest-rate smoothing central banker. It would be interesting to consider the result in the light of a completely backward-looking Phillips curve model, but that would not be in the spirit of the class of New Keynesian models emphasized here.

4.5. Optimal policy inertia and dynamics

We now analyze the stabilization bias problem in terms of the magnitude, direction and persistence of dynamic adjustments in the model. Consider a positive domestic technology shock. This is shown in Fig. 1. The solid lines correspond to the case when optimal policy is conducted with the supposed pre-commitment to society's loss function (COM). The lines marked with crosses describe the equilibrium under the optimally delegated central banker with no smoothing behavior (DEL1). Finally the circled lines represent discretionary policy under optimal delegation with an interest-rate smoothing central bank (DEL2).

Consider first the outcome under the regime COM. Generally, the amplitude of the impulse responses under this case are much smaller than both cases of discretion. Given a positive domestic technology shock, the direct effect through the production function should increase the level of output gap. However, for a given level of nominal interest rate, the technology shock lowers the natural interest rate, resulting in a larger gap between the nominal and natural interest rates. This has a tendency to depress the output gap initially. This can be seen by inspecting the IS equation in (4). Since policy responds to output-gap deviations, the nominal interest rate falls, and this creates a positive output gap eventually. Given a fall in the nominal interest rate, there is a currency depreciation resulting in the nominal exchange rate deviation being positive under uncovered interest rate parity. Alternatively, one can observe from (7) that a positive technology shock has the Samuelson-Balassa effect of improving the small open economy's terms of trade and therefore creating a depreciation of its currency. A depreciation of the domestic currency which is persistent, creates an expectation that future imports prices will be falling as demand switches from imports to domestic goods. This causes domestic inflation to rise boosted by the rise in output gap, while imports inflation falls negating the tendency of a depreciation to create a positive law-of-one-price (LOP) gap. In fact a negative LOP

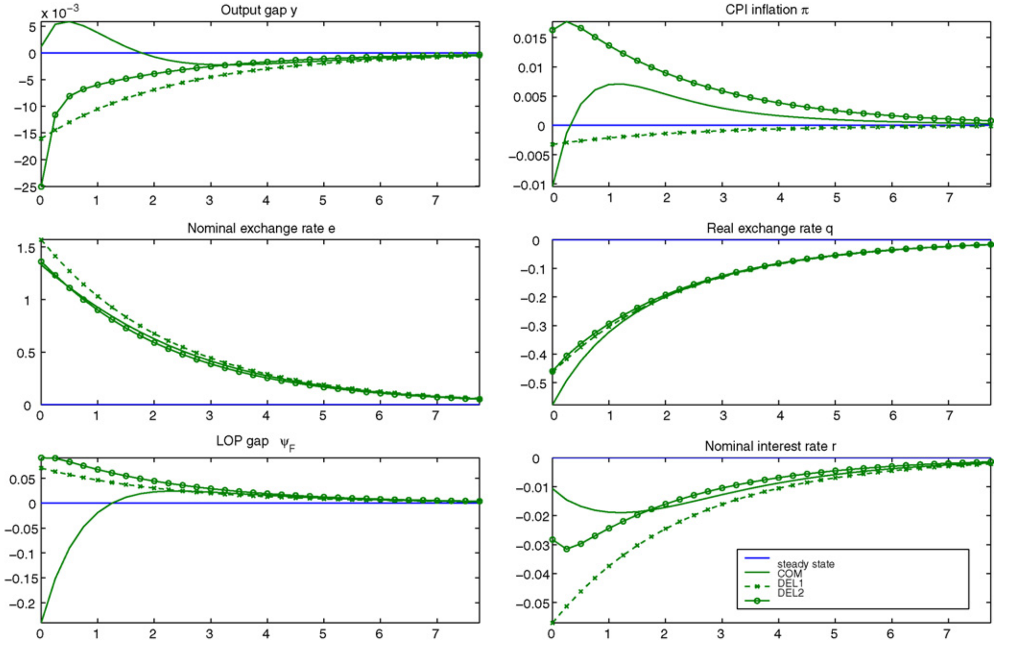

Fig. 2. Impulse responses to foreign technology shock: benchmark economy.

gap is obtained which reinforces the fall in imports inflation. This can be verified by inspecting Eqs. (3)–(6).

When the optimally chosen central bank in regime DEL1 has the incentive to "cheat" under discretion, and this is anticipated by the private agents, it ends up allowing for a smaller positive effect of a domestic technology increase on output gap, but, a larger negative effect on inflation, compared to the commitment benchmark. Part of this is also because under discretion, there is a bias toward stabilizing the LOP gap which has a trade-off effect on output gap and inflation, in contrast to the commitment case.

However, when one considers allowing for the case of optimally delegating discretionary policy to a policy maker who has the additional preference for interest-rate smoothing (DEL2), it can be seen that the impulse responses tend to track those of the outcomes under commitment much better. Here the discretionary interest-rate smoother foregoes some of the bias toward output gap or LOP gap stabilization. This is consistent with the result shown in Section 3.4 where we showed that the explicit interest-rate smoothing objective results in a weakening of the equilibrium monetary-policy trade-off between inflation and output gap. This is a desirable property if the central bank were to act with discretion.

The same can be seen when we consider the case of an exogenous technology shock in the rest of world, in Fig. 2. Thus, while pure discretionary policy without interest-rate smoothing carries a large trade off between stabilizing domestic inflation, output gap, interest rate and the LOP gap, optimally delegating policy to an interest-rate smoother is seen to dampen such a trade off by bringing the equilibrium dynamics of the economy closer to that under the desired commitment outcome.

5. Conclusion

In this paper, the role of optimal interest-rate smoothing is considered in the context of a small open economy model that exhibits an endogenous monetary-policy trade-off between inflation and output gap, as a result of imperfect exchange rate pass through to domestic prices. It was shown that the Woodford (1999) conclusion about optimal monetary policy inertia still carries through in the small-open-economy setting that breaks the isomorphism between the closed- and open-economy monetary policy models. The paper proceeded from the benchmark of assuming that the central bank can solve a pre-commitment policy problem. However, if the central bank cannot commit to that policy and thus acts in discretion, this creates too much stabilization on the central bank's part. Such a stabilization bias manifested itself in the form of greater uncertainty around the macroeconomic variables in the model. The bias is also measured as a larger loss in terms of the social loss function.

A possible solution, as was proposed by Woodford (1999), is to hire a central banker whose preferences include interest-rate smoothing even though this is not shared by society's preferences. It was shown in the paper that allowing for interest-rate smoothing under discretion results in a difference rule for the interest rate. That is, optimal (discretionary) policy in such a case involves setting the change in interest rate in response to CPI inflation and output gap. The reason for having an interest-rate smoothing central banker is that it tends to introduce intrinsic inertia into the interest rate process, thus approximating the desired commitment policy. We showed analytically that the effect of this is to force history dependence in policy decisions under discretion, and that this acts to soften the equilibrium endogenous monetary policy trade-off. This result was verified in various numerical examples.

Finally, there is no reason to accept the assumption of a central bank committing to an ex-ante optimal policy plan without explicit modeling of incentives that enforce commitment. If so, then it may well be that observed strong and positive autocorrelation in small-open-economy nominal interest rates could be explained by an optimal social delegation of discretionary policy to an interest-rate smoothing central bank. Such a hypothesis could potentially be empirically tested. For example, the Bayesian structural estimation approach in Kam, Lees, and Liu (2008) could be used, and posterior-odds model comparisons can be made across alternative policy regimes to determine which assumption on policy regimes is more probable, given the data for each small open economy. We leave this suggestion to future explorers.

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Appendix A. The private sector model

The private sector of the model consists of the household sector, imperfectly competitive domestic goods firms, foreign goods importers, the central bank, and exogenous processes for the rest of the world.

A.1. Household sector

The representative household maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] := \sum_{t=0}^{\infty} \beta^t \int_{\mathcal{B}(Z)} \left[\frac{[C_t(\mathbf{z}_t)]^{1-\sigma}}{1-\sigma} - \frac{[N_t(\mathbf{z}_t)]^{1+\varphi}}{1+\varphi} \right] \mu_t(\mathbf{z}_0, \mathrm{d}\mathbf{z}_t), \tag{38}$$

subject to the sequence of constraints given by

$$\int_0^1 [P_{H,t}(i) C_{H,t}(i) + P_{F,t}(i) C_{F,t}(i)]\, \mathrm{d}i + \mathbb{E}_t Q_{t,t+1} B_{t+1} \leq B_t + W_t N_t + T_t; \quad t \in \mathbb{N}, \tag{39}$$

where $\mathcal{B}(Z)$ is the Borel sigmafield generated by the compact cube $Z \ni \mathbf{z}_t$ and μ_t is the time-t probability measure on $\mathcal{B}(Z)$. The prices of home and foreign goods of type i are respectively given by $P_{H,t}(i)$ and

$P_{F,t}(i)$, B_{t+1} is the nominal value of assets held at the end of period t, W_tN_t is the total wage income and T_t is a lump-sum tax or transfer. The stochastic discount factor is $Q_{t,t+1}$. The consumption index C_t is linked to a continuum of domestic, $C_{H,t}(i)$, and foreign goods, $C_{F,t}(i)$ which exist on the interval of [0, 1] through the following indexes

$$C_t = [(1-\gamma)^{1/\eta} C_{H,t}^{((\eta-1)/\eta)} + \gamma^{1/\eta} C_{F,t}^{((\eta-1)/\eta)}]^{(\eta/(\eta-1))} \tag{40}$$

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{((\varepsilon-1)/\varepsilon)}\,\mathrm{d}i\right]^{\varepsilon/(\varepsilon-1)}, \quad C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{((\varepsilon-1)/\varepsilon)}\,\mathrm{d}i\right]^{\varepsilon/(\varepsilon-1)}. \tag{41}$$

Thus the elasticity of substitution between home and foreign goods is given by $\eta > 0$ and the elasticity of substitution between goods within each goods category (home and foreign) is $\varepsilon > 0$. Optimal allocation of the household expenditure across each good type gives rise to the demand functions:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}, \quad C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t} \tag{42}$$

for all $i \in [0, 1]$ where $P_{j,t} = \left(\int_0^1 P_{j,t}(i)^{1-\varepsilon}\,\mathrm{d}i\right)^{(1/(1-\varepsilon))}$,$j = H, F$ and $C_{H,t} = (1-\gamma)((P_{H,t}/P_t))^{-\eta} C_t$, and $C_{F,t} = \gamma((P_{F,t}/P_t))^{-\eta} C_t$. The consumer price index can be solved as $P_t = [(1-\gamma)P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta}]^{(1/(1-\eta))}$. Another intratemporal condition relating labor supply to the real wage must also be satisfied

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{43}$$

Finally, intertemporal optimality for the household decision problem must satisfy the stochastic Euler equation

$$\beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \right\} = Q_{t,t+1} \tag{44}$$

where $Q_{t,t+1}$ is the stochastic discount factor.

A.2. Domestic production

There is a continuum of monopolistically competitive firms defined on [0, 1]. Firms utilize a constant returns-to-scale technology, $Y_t(i) = Z_tN_t(i)$, where $Z_t = \exp(z_t)$ is a total productivity shifter. Cost minimization leads to the first-order condition

$$MC_{H,t}(i)Z_t = W_t \tag{45}$$

Given (45) it can be seen that nominal marginal cost is common for all firms such that $MC_{H,t}(i) = MC_{H,t}$ for all $i \in [0, 1]$. In our analysis on optimal monetary policy, it is assumed that fiscal policy provides for an employment subsidy of τ to deliver the first-best allocation under flexible prices. Therefore, (45) can be rewritten, after integrating across all firms, as

$$mc_{H,t} = \frac{(1-\tau)W_t}{Z_tP_{H,t}}. \tag{46}$$

A.3. Domestic pricing

The retail side of the firms producing domestic goods change prices according to a discrete-time version of the Calvo (1983) model. The signal for a price change is a stochastic time-dependent process governed by a geometric distribution. In a symmetric equilibrium all firms that get to set their price in the same period choose the same price. Thus prices evolve according to

$$P_{H,t} = [(1-\theta_H)(P_{H,t}^{new})^{1-\varepsilon} + \theta_H(P_{H,t-1})^{1-\varepsilon}]^{(1/(1-\varepsilon))}. \tag{47}$$

Thus, when setting $P_{H,t}^{new}$, each firm will seek to maximize the value of expected discounted profits:

$$\max_{\{P_{H,t}^{new}\}_{t\in\mathbb{N}}} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} Q_{t,t+k}\theta_H^k [P_{H,t}^{new} - MC_{H,t+k}(i)] C_{H,t+k}(P_{H,t}^{new}, i) \right\} \tag{48}$$

subject to $C_{H,t+k}(i) = (P_{H,t}^{new}/P_{H,t+k})^{-\varepsilon} C_{H,t+k}$. The optimal pricing strategy is thus one of choosing an optimal path of price markups as a function of rational expectations forecast of future demand and marginal cost conditions,

$$P_{H,t}^{new} = P_{H,t}\left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{\mathbb{E}_t \sum_{k=0}^{\infty} Q_{t,t+k}\theta_H^k (P_{H,t}/P_{H,t+k})^{-1-\varepsilon} (MC_{H,t+k}/P_{H,t+k}) C_{H,t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} Q_{t,t+k}\theta_H^k (P_{H,t}/P_{H,t+k})^{-\varepsilon} C_{H,t+k}}. \tag{49}$$

Notice that if the chance for stickiness in price setting is nil, $\theta_H = 0$ for all $k \in \mathbb{N}$, the first order condition in (49) reduces to $mc_{H,t} = (1 - \varepsilon^{-1})$, for all t, which says that the optimal price is a constant markup over marginal cost, or that the real marginal cost is constant over time. This is the same result as that for a static model of a firm with monopoly power. Log-linearizing the pricing decision and straightforward algebra produce the NK Phillips curve for domestic goods:

$$\pi_{H,t} = \beta \mathbb{E}_t\{\pi_{H,t+1}\} + \lambda_H mc_{H,t}, \tag{50}$$

where $\lambda_H = \theta_H^{-1}(1-\theta_H)(1-\beta\theta_H)$.

A.4. Imports retailer

Let ϵ_t denote the level of the nominal exchange rate. There exists local firms acting as retailers who purchase imports at the marginal cost equal to the imports price in domestic dollar terms, $\epsilon_t P_{F,t}^*(j)$, and re-sell them domestically at a markup price, $P_{F,t}^{new}$. It is the stickiness in the domestic price of imported goods that will cause a persistent and potentially large gap in what would otherwise be the law of one price. Thus the local retailer importing good j solves

$$\max_{\{P_{F,t}^{new}\}_{t\in\mathbb{N}}} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} Q_{t,t+k}\theta_F^k [P_{F,t}^{new} - \epsilon_{t+k} P_{F,t+k}^*(j)] C_{F,t+k}(j) \right\} \tag{51}$$

such that $C_{F,t+k}(j) = (P_{F,t}^{new}/P_{F,t+k})^{-\varepsilon} C_{F,t+k}$. The optimal pricing strategy is thus

$$P_{F,t}^{new} = P_{F,t}\left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{\mathbb{E}_t \sum_{k=0}^{\infty} Q_{t,t+k}\theta_F^k (P_{F,t}/P_{F,t+k})^{-1-\varepsilon} ((\epsilon_{t+k} P_{F,t+k}^*)/P_{F,t+k}) C_{F,t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} Q_{t,t+k}\theta_F^k (P_{F,t}/P_{F,t+k})^{-\varepsilon} C_{F,t+k}}$$

given the evolution of the aggregate retail imports price index as

$$P_{F,t} = [(1-\theta_F)(P_{F,t}^{new})^{1-\varepsilon} + \theta_F (P_{F,t-1})^{1-\varepsilon}]^{(1/(1-\varepsilon))}.$$

Let e_t, $p_{F,t}^*$ and $p_{F,t}$ denote the log deviations of the nominal exchange rate, foreign price of imports and domestic retail price of imports respectively. The law-of-one-price gap in log-deviation terms is measured as

$$\psi_{F,t} = e_t + p_{F,t}^* - p_{F,t}. \tag{52}$$

A first-order approximation to the pricing dynamics will result in a similar aggregate supply schedule

$$\pi_{F,t} = \beta \mathbb{E}_t\{\pi_{F,t+1}\} + \lambda_F \psi_{F,t}, \tag{53}$$

where $\lambda_F = \theta_F^{-1}(1-\theta_F)(1-\beta\theta_F)$. Notice that if the domestic dollar price of foreign goods exceed the domestic retail price of foreign goods, or $\psi_{F,t} > 0$, ceteris paribus, $\pi_{F,t} > 0$.

A.5. Market clearing conditions

In the rest of the world, it is assumed that in the limit of being a closed economy, the home goods price of the rest of the world equals its CPI, or $P^*_{H,t} = P^*_t$ and consumption equals output, $C^*_t = Y^*_t$. Market clearing in the small open economy requires that

$$Y_t(i) = C_{H,t}(i) + C^*_{H,t}(i) \tag{54}$$

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left[\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}(1-\gamma)C_t + \left(\frac{P_{H,t}}{\epsilon_t P^*_t}\right)^{-\eta}\gamma^* Y^*_t\right] \tag{55}$$

A.6. Dynamics and policy in the rest of the world

The rest of the world maintains a first-best flexible price equilibrium. Specifically the aggregate supply equivalent of (50) in the rest of the world, combining with labor supply decisions, yields $mc^*_t = (\sigma+\varphi)y^*_t - (1+\varphi)z^*_t$, and under the natural flexible price level of output in the world economy, $mc^*_t = 0$, implying $\pi^*_t = 0$, for all t. Thus output in the rest of the world equals its natural output

$$y^*_t = \left(\frac{1+\varphi}{\sigma+\varphi}\right) z^*_t. \tag{56}$$

From the IS equation for the rest of the world,

$$y^*_t = \mathbb{E}_t y^*_{t+1} - \frac{1}{\sigma}(r^*_t - \mathbb{E}_t \pi^*_{t+1}) = \rho\left(\frac{1+\varphi}{\sigma+\varphi}\right) z^*_t - \frac{1}{\sigma} r^*_t \tag{57}$$

making use of (9) and (56) in (57) yields the natural rate of interest in the rest of the world as

$$r^*_t = -\sigma\left(\frac{1+\varphi}{\sigma+\varphi}\right)(1-\rho) z^*_t. \tag{58}$$

Appendix B. Proof of Proposition 1

The system of forward-looking private-sector variables (1)–(6) together with the predetermined Lagrange multipliers can be written in the canonical form

$$\begin{pmatrix} \mathbb{E}_t \mathbf{x}_{t+1} \\ \phi_t \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \mathbf{x}_t \\ \phi_{t-1} \end{pmatrix} + \begin{pmatrix} \mathbf{F} \\ \mathbf{0}_4 \end{pmatrix} \mathbf{z}_t \tag{59}$$

where $\mathbf{x}_t = (\pi_{H,t} \quad \tilde{y}_t \quad \pi_{F,t} \quad \psi_{F,t})'$, $\phi_t = (\phi_{1,t} \quad \phi_{2,t} \quad \phi_{3,t} \quad \phi_{4,t})'$, and $\mathbf{z}_t = (z^*_t \quad z_t)'$. In a rational expectations (RE) equilibrium given by Definition 1, the unique and bounded solution for the forward-looking part of the model can be found by "solving forward" to yield

$$\mathbf{x}_t = \mathbf{G}\phi_{t-1} - \sum_{j=0}^{\infty} \mathbf{H}^{-(j+1)} \mathbf{F} \mathbb{E}_t \mathbf{z}_{t+j} \tag{60}$$

where $\mathbf{G}$ and $\mathbf{H}$ contain coefficients that are determined in the RE equilibrium. The predetermined Lagrange multipliers can be solved backward to obtain

$$\phi_t = \mathbf{C}\mathbf{x}_t + \mathbf{D}\phi_{t-1} - \mathbf{N}\phi_{t-1} - \mathbf{C}\sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F}\mathbb{E}_t\mathbf{z}_{t+j} \tag{61}$$

where $\mathbf{N} = \mathbf{CG} + \mathbf{D}$. By recursive backward substitution of (61), and given the initial conditions $\phi_{-1} = \mathbf{0}_{4\times 1}$, this can be written as

$$\phi_t = -\sum_{s=0}^{t}\mathbf{N}^s\left(\mathbf{C}\sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F}E_{t-s}\mathbf{z}_{t-s+j}\right). \tag{62}$$

From the first-order condition (19) of the central bank's problem, we have $b_r(r_t - r^*) + (\omega_s/\sigma)\phi_{2,t} - \phi_{4,t} = 0$ which, making use of the solution (61) and lagging (19) by one period, can be re-written as

$$b_r(r_t - r^*) = \begin{pmatrix} 1 \\ -\dfrac{\omega_s}{\sigma} \end{pmatrix}' \left\{ \begin{bmatrix} N_{21}\phi_{1,t-1} + N_{22}\phi_{2,t-1} + N_{23}\phi_{3,t-1} + N_{24}\left(b_r(r_{t-1} - r^*) + \dfrac{\omega_s}{\sigma}\phi_{2,t-1}\right) \\ N_{41}\phi_{1,t-1} + N_{42}\phi_{2,t-1} + N_{43}\phi_{3,t-1} + N_{44}\left(b_r(r_{t-1} - r^*) + \dfrac{\omega_s}{\sigma}\phi_{2,t-1}\right) \end{bmatrix} + \begin{pmatrix} \mathbf{C}_2 \\ \mathbf{C}_4 \end{pmatrix} \right\}$$
$$\times \sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F}\mathbb{E}_t\mathbf{z}_{t+j},$$

where N_{ij} refers to the (i,j) th element of the matrix $\mathbf{N}$ and $\mathbf{C}_i$ refers to the ith row of matrix $\mathbf{C}$. Using (62) we can re-write this as

$$(r_t - r^*) = \rho_r(r_{t-1} - r^*) - \mathbf{\Theta}_b\sum_{s=0}^{t-1}\mathbf{N}^s\mathbf{C}\sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F}E_{t-s}\mathbf{z}_{t+j-s-1} - \mathbf{\Theta}_f\sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F}\mathbb{E}_t\mathbf{z}_{t+j}$$

where

$$\rho_r = N_{44} - \frac{\omega_s}{\sigma}N_{24},$$
$$\mathbf{\Theta}_b = b_r^{-1}\begin{pmatrix} 1 \\ -\dfrac{\omega_s}{\sigma} \end{pmatrix}' \begin{pmatrix} N_{21} & \left(N_{22} + \dfrac{\omega_s}{\sigma}N_{24}\right) & N_{23} \\ N_{41} & \left(N_{42} + \dfrac{\omega_s}{\sigma}N_{44}\right) & N_{43} \end{pmatrix} (\,\mathbf{I}_3 \quad \mathbf{0}_{3\times 1}\,),$$
$$\mathbf{\Theta}_f = b_r^{-1}\begin{pmatrix} 1 \\ -\dfrac{\omega_s}{\sigma} \end{pmatrix}' \begin{pmatrix} \mathbf{C}_2 \\ \mathbf{C}_4 \end{pmatrix}.$$

In the model, it was assumed that $\mathbf{z}_t$ follows a first-order Markov process given by the transition matrix $\mathbf{M}$ where $\mathbf{M}$ is a stable matrix. Therefore, the interest rate process can be further solved in terms of the primitive shocks as (22), given below:

$$r_t = (1 - \rho_r)r^* + \rho_r r_{t-1} - \mathbf{\Theta}_b\sum_{s=0}^{t-1}\mathbf{N}^s\mathbf{C}\sum_{j=0}^{\infty}H^{-(j+1)}\mathbf{F}\mathbf{M}^j\mathbf{z}_{t-s-1} - \mathbf{\Theta}_f\sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F}\mathbf{M}^j\mathbf{z}_t.$$

The optimal interest-rate process under the pre-commitment policy results in an inertial rule so long as $N_{44} - (\omega_s/\sigma)N_{24} \neq 0$. This rule is both forward and backward looking in terms of past and current forecasts of foreign and domestic technology shocks, since $\mathbf{\Theta}_b \neq 0$ and $\mathbf{\Theta}_f \neq 0$. Finally, since the coefficient on lag interest rate, ρ_r, is independent of $\mathbf{M}$, the inertia in policy under pre-commitment in this small open-economy-model is not an artefact of serial correlation in the exogenous stochastic processes $\mathbf{z}_t$.

Appendix C. Hybrid Phillips curves

As an alternative to the purely forward-looking Phillips curve model, we apply the approach of Amato and Laubach (2003) in generalizing the New-Keynesian Phillips curve for the domestic and imported goods sectors. The modification essentially allows for a fraction of domestic and foreign goods firms to set prices as a function of their own historical prices. Specifically, we have the following assumption. Suppose now, at the beginning of each period, nature draws from fixed a geometric distribution and determines with probability $\lambda_r^j \in (0, 1)$, and $j = H, F$, that a firm will get to set price according to the Calvo model, so that the optimal price for domestic and foreign goods firms will be, respectively:

$$P_{F,t}^{opt} = P_{F,t}\left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{\mathbb{E}_t \sum_{k=0}^{\infty} Q_{t,t+k}\theta_F^k (P_{F,t}/P_{F,t+k})^{-1-\varepsilon}((\epsilon_{t+k}P_{F,t+k}^*)/P_{F,t+k})C_{F,t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} Q_{t,t+k}\theta_F^k (P_{F,t}/P_{F,t+k})^{-\varepsilon} C_{F,t+k}} \tag{63}$$

$$P_{H,t}^{opt} = P_{H,t}\left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{\mathbb{E}_t \sum_{k=0}^{\infty} Q_{t,t+k}\theta_H^k (P_{H,t}/P_{H,t+k})^{-1-\varepsilon}(MC_{H,t+k}/P_{H,t+k})C_{H,t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} Q_{t,t+k}\theta_H^k (P_{H,t}/P_{H,t+k})^{-\varepsilon} C_{H,t+k}} \tag{64}$$

Otherwise each type respectively will end up, with probability $1 - \lambda_r^j$, setting a price according to the rule of thumb: $P_{j,t}^{rot} = P_{j,t-1}^{new}(P_{j,t-1}/P_{j,t-2})$; $j = H, F$. This basically says that rule-of-thumb price setters will base their current pricing strategy on last period's aggregate price, $P_{j,t-1}^{new}$, which includes forward-looking and backward-looking prices, times gross inflation between two periods. We assume that the new aggregate price is given by the index

$$P_{j,t}^{new} = [(1 - \lambda_r^j)(P_{j,t}^{rot})^{1-\varepsilon} + \lambda_r^j (P_{j,t}^{opt})^{1-\varepsilon}]^{(1/(1-\varepsilon))} \tag{65}$$

Some algebraic manipulation will produce the more general New-Keynesian Phillips curve for domestic and foreign goods, respectively, as (36) and (37).

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