Incomplete Risk Sharing and Monetary Policy in a Small Open Economy∗

Jaime Alonso-Carrera† and Timothy Kam∗

ABSTRACT

We propose an alternative and simple small open economy model with incomplete international asset markets. This model allows us to analytically dissect the role of market incompleteness in a small open economy model of monetary policy. Our model nests the canonical two-equation closed economy and the complete-markets small open economy as special cases. In contrast to existing models, we show that asset market incompleteness results in the real exchange rate being an explicit, or irreducible, variable in the equilibrium characterization. Moreover, this explicit equilibrium exchange rate channel has an endogenous “cost-push” monetary-policy trade-off interpretation. This provides an alternative means of breaking the “monetary-policy isomorphism” between the small open economy and its closed economy limit. We then show that established lessons on local stability of rational expectations equilibrium under alternative monetary policies are reversed when the economy cannot completely insure country-specific risks.

JEL CODES: E52; F41

KEYWORDS: Incomplete Markets; Monetary Policy Isomorphism; Exchange Rate; Equilibrium Determinacy

†Departamento de Fundamentos del Análisis Económico & RGEA
Universidade de Vigo
Campus As Lagoas-Marcosende
36310 Vigo, Spain
Email: jalonso@uvigo.es

‡School of Economics
H.W. Arndt Building 25a
The Australian National University
A.C.T. 0200, Australia
E-mail: timothy.kam@anu.edu.au

∗J. Alonso-Carrera thanks the School of Economics and CAMA at the Australian National University for their hospitality during a visit where this paper was written. Financial support from the Spanish Ministry of Science and FEDER through grant ECO2008-02752; from the Spanish Ministry of Education through grant PR2009-0162; from Xunta de Galicia through grant 10PXIB30001777P; from the Generalitat de Catalunya through grant SGR2009-1051; from EDN program funded by Australian Research Council; and from CAMA at Australian National University are gratefully acknowledged. T. Kam thanks RGEA at Universidade de Vigo for funding support and their hospitality. We thank Craig Burnside, V.V. Chari, Richard Dennis, Mark Gertler, Simon Gilchrist, Bruce Preston, Christoph Thoenissen and Tao Zha for useful suggestions and beneficial conversations. This paper evolved from an earlier version prepared for the Northwestern CIED and RBNZ Monetary Policy Conference, “Twenty Years of Inflation Targeting” held in Wellington in December 2009.
1 Introduction

Why should small open economy monetary authorities care about international exchange rates? Is there a justification for managing exchange rates, and what is its connection to incomplete international risk sharing of country-specific shocks? In practice, in many small open economies with floating exchange rate regimes, the dynamics of the exchange rate matter, in structural modelling, and for monetary policy design. Also it remains unclear in the literature, which monetary policy can induce equilibrium stability, when the dynamics of the exchange rate cannot be decoupled from inflation and output gap in an equilibrium characterization.

In standard monetary-policy small open economy models, the exchange rate is a reducible variable in equilibrium. In other words, its explicit dynamics can be decoupled from necessary equilibrium conditions. Specifically, under certain restrictions on inter- and intra-temporal elasticities of substitution, the open economy dimension merely alters the equilibrium conditions that are familiar to a closed economy model in terms of the slopes of an IS curve and a Phillips curve [see Benigno and Benigno, 2003; Gali and Monacelli, 2005; Clarida et al., 2001]. More generally, if these parametric restrictions are relaxed, Benigno and Benigno [2003] have shown that the monetary policy implication for the open economy is no longer isomorphic to its closed-economy limit. That is, the design of monetary policy for the small open economy must also take into account the trade-offs arising from the open economy channels. However, the explicit dynamics of the exchange rate is still redundant in these systems as long as the open economy has access to a complete international state-contingent asset market.

We propose an alternative and tractable small open economy model with incomplete international asset markets in order to address these two questions. Our model nests the canonical complete-markets small open economy model of Gali and Monacelli [2005] and the standard New Keynesian closed economy model [see e.g. Woodford, 2003] as special cases. Our contribution is two-fold.

First, incomplete markets result in an irreducible and explicit exchange rate channel, in the model’s equilibrium characterization. This result introduces a truly endogenous “cost-push” trade-off for monetary policy, in the sense that this “cost-push” term is itself an irreducible endogenous variable. This is in contrast to complete-markets small open economy models, where the reducible exchange-rate channel merely creates a “cost-push” trade-off in terms of exogenous factors such as foreign output gap. We also show that the irreducibility of the exchange rate and cost-push trade-off depends solely on an assumption of incomplete international asset markets.\(^1\) As a corollary, we also obtain a break in the “monetary-policy isomorphism” between the small open economy and its closed economy limit.

Second, we show that established lessons on local stability of rational expectations equilibrium (REE) under alternative monetary policies are reversed as a result of the fact that the economy cannot completely insure country-specific risks completely. The latter poses additional restrictions on the admissibility of policy rules in inducing determinate REE. We show that while the

\(^1\)Our general model admits two other sources through which the exchange rate may explicitly matter: (i) The interaction between endogenous discounting—which is required to induce a unique steady state [see Schmitt-Grohé and Uribe, 2003] —and intra-temporal substitution between home- and foreign-produced final consumption goods; and (ii) The possibility of an imported input in the small economy’s production structure. We show that in the limit when these two sources are removed, an irreducible exchange rate dynamics still remains; and this is purely a result from the existence of incomplete international asset markets.
inability of a small open economy to insure its country-specific technology risk reduces such admissible sets of monetary policies, it can be improved by a family of simple policies that take into account exchange rate growth as well.

We provide a simple theoretical foundation for standard monetary policy modelling and practice in small open economies with floating exchange rates. In practice, modellers and policymakers in these economies take into account explicit exchange rate dynamics, in model equilibrium conditions, and, also in policy objectives. For example, clause 4(b) of New Zealand’s 2002 Policy Targets Agreement states that:

“[I]n pursuing its price stability objective, the Bank shall seek to avoid unnecessary instability in output, interest rates and the exchange rate”.

There are existing departures from the “isomorphism” result, in the sense that the exchange rate is irreducible from equilibrium characterization. A seminal departure is Monacelli [2005], who assumes complete asset markets and sticky prices in the imported goods sector. The latter creates a short-run law of one price gap, and therefore imperfect pass through of the exchange rate to domestic prices. This mechanism results in an endogenous cost-push shock to the Phillips curve. de Paoli [2009b] also considers a small open economy with incomplete markets. de Paoli [2009b] ensures the existence of a well-defined steady state by introducing portfolio adjustment cost for trade in the non-state-contingent money claims. de Paoli [2009b] focuses on business cycle and welfare properties of her model with international asset-market incompleteness. In contrast, by assuming negligibly small endogenous discounting of future payoffs of agents in our model, we are able to have a more tractable model and we explicitly characterize the channels that deliver our endogenous monetary-policy trade-off. Moreover, we do not require additional price-stickiness assumptions on imported goods as in Monacelli [2005], in order to deliver irreducible exchange rate dynamics in equilibrium.

Our analysis in this paper also complements existing two-country open economy models with incomplete asset markets. For example, Corsetti et al. [2010] show that international asset market incompleteness restrict open economies in achieving efficient consumption allocations, regardless of the existence of price flexibility. However, the issue of closing the small open economy with incomplete markets is also important, and this is a non-issue in large two-country models. Moreover, the role of international asset market incompleteness in affecting REE determinacy or indeterminacy under alternative monetary-policy rules have not been studied, in either the two-country or small open economy environments. This paper deals with these issues for the small open economy case.

Therefore, our contribution is to fill a gap in the literature by providing a tractable version of a small-open economy model whose equilibrium characterization, allows for a careful dissection of the role of incomplete markets in delivering an endogenous output-gap-inflation trade off. Moreover, our application provides a complementary exercise, with respect to open-economy models with incomplete markets [e.g. de Paoli, 2009b; Corsetti et al., 2010], by revisiting and contrasting with well-known results [e.g. Bullard and Mitra, 2002; Llosa and Tuesta, 2008] in terms of indeterminacy of REE vis-à-vis standard simple monetary policy rules.

2The Reserve Bank of New Zealand pioneered inflation targeting, implementing this policy in 1990.
The rest of the paper is organized as follows. In Section 2, we describe the details of our alternative model. Then we characterize competitive equilibrium in Section 3 and explain the effect of how asset market incompleteness in our model results in an endogenous monetary policy trade off in Section 3.3. We then parametrize the model in Section 4. Here we will discuss the implication of market incompleteness on the reduced-form parameters describing the equilibrium trade offs. In Section 5, we analyze the implications of market incompleteness—and therefore the additional restrictions on stability-inducing monetary policy rules—on equilibrium determinacy. Finally in Section 6, we conclude.

2 Model

We propose a small open-economy model consisting of monopolistically competitive domestic goods markets with nominal pricing rigidity, and, households that only have access to a restricted set of internationally traded non-state-contingent assets – viz. the incomplete international asset markets assumption. The domestic economy is small in the sense that local equilibrium outcomes here do not have any impact on the rest of the world, but, the converse is not true. The foreign economy (or the rest of the world) is treated as a large closed economy. We will use variables with an asterisked superscript (e.g. \(X^*\)) to refer to the foreign country and variables without an asterisk to denote the small domestic economy. Subscripts “H” (for Home) and “F” (for Foreign) on certain variables will denote the country of origin for quantities and their supporting prices.

A special case of our model is a version of the complete-markets model similar in form to Gali and Monacelli [2005] or Clarida et al. [2001]. The notion of trade openness in our model admits: (i) imports of intermediate inputs for producing a final good in the small economy, along the lines of McCallum and Nelson [1999] and Chari et al. [2002]; and (ii) trade in the final differentiated goods. If we shut down these open-economy features in our model, we will also obtain the familiar limit of the two-equation closed economy New-Keynesian model similar to that discussed in Woodford [2003].

2.1 Representative household

As in McCallum and Nelson [1999] or Benigno and Thoenissen [2008], individuals in our small open-economy have access only to a pair of domestic and foreign nominal uncontingent bonds denominated in their own currencies, respectively, \(B_t\) and \(B^{*\prime}_t\). More precisely, let \(h^t := (z_0, ..., z_t)\) denote the \(t\)-history of aggregate shocks, where \(z_t = (A_t, Y^{*\prime}_t)\) is a vector of domestic productivity and foreign output levels, respectively. \(B_{t+1}(h^t)\) or \(B^{*\prime}_{t+1}(h^t)\) denotes a claim on one unit of currency following \(h^t\), and is independent of any continuation state \(z_{t+1}\) that may occur at \(t + 1\). Let \(S_t(h^t)\) denote the nominal exchange rate, defined as the domestic currency price of a unit of foreign currency. In domestic currency terms, the prices of one unit of the nominal bonds \(B_{t+1}(h^t)\) and \(B^{*\prime}_{t+1}(h^t)\) are, respectively, \(1/[1 + r_t(h^t)]\) and \(S_t(h^t)/[1 + r_t^*(h^t)]\), where \(r_t\) and \(r_t^*\) are the respective domestic and the foreign nominal interest rates.

The representative consumer in the domestic country faces the following sequential budget
constraint, for each \( t \in \mathbb{N} \), and each (measurable) history \( h^t \),

\[
P_t (h^t) C_t (h^t) + \frac{B_{t+1} (h^t)}{1+r^t (h^t)} + \frac{S_t (h^t) B_{t+1}^* (h^t)}{1+r^t (h^t)} \leq W_t (h^t) N_t (h^t) + B_t (h^{t-1}) + S_t (h^t) B_t^* (h^{t-1}) + \Pi_t (h^t),
\]

(2.1)

where \( P_t \) is the domestic consumer price indexes, \( C_t \) is a composite consumption index, \( W_t \) is the nominal wage rate, \( N_t \) denotes the hours of labor supplied, and, \( \Pi_t \) is the total nominal dividends received by the consumer from holding equal shares of the domestic firms.

A minor difference of our model to Galí and Monacelli [2005] and McCallum and Nelson [1999] is that consumers exhibit an endogenous discount factor that we denote by \( \rho_t \). This assumption is introduced in order to ensure a unique nonstochastic steady-state consumption level, following Schmitt-Grohé and Uribe [2003].\(^3\) However, this is not a fundamental assumption for our conclusions with respect to the endogenous monetary-policy trade off arising from the real-exchange-rate channel.\(^4\) The consumers’ preferences are given by the following present-value total expected utility function:

\[
\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \rho_t \left\{ U [C_t (h^t)] - V [N_t (h^t)] \right\} \right\}, \quad \rho_t = \begin{cases} \beta \left( C_{t-1}^u (h^{t-1}) \right) \rho_{t-1} & \text{for } t > 0, \\ 1 & \text{for } t = 0, \end{cases}
\]

(2.2)

where \( \mathbb{E}_0 \) denotes the expectations operator conditional on time-0 information, and, \( C_t^0 \) denotes the cross-economy average level of consumption. For concreteness, we will consider the following parametric form for the function \( \beta : \mathbb{R}_+ \to (0, 1) \), following Ferrero et al. [2007]:

\[
\hat{\beta}(C_t^0) = \frac{\hat{\beta}}{1 + \phi (\ln C_t^0 - \bar{\theta})}; \quad \hat{\beta} \in (0, 1).
\]

(2.3)

We do not impose a priori any condition on the sign of the dependence of the discount factor on average consumption, i.e., we only assume that \( \hat{\beta}'(C_t^0) \neq 0 \). We also assume that per-period utility of consumption and labor have the respective forms: \( U [C_t (h^t)] = C_t (h^t)^{1-\sigma} / (1-\sigma) \), and, \( V [N_t (h^t)] = \psi N_t (h^t)^{1+\varphi} / (1+\varphi) \), where \( \sigma > 0, \varphi > 0 \), and \( \psi > 0 \).

The household chooses an optimal plan \( \{ C_t (h^t), N_t (h^t), B_{t+1} (h^t), B_{t+1}^* (h^t) \}_{t \in \mathbb{N}} \) to maximize (2.2) subject to (2.1). Unilaterally, the household will take the aggregate outcome \( C_t^0 (h^t) \), nominal prices \( \{ W_t (h^t), P_t (h^t), S_t (h^t) \}_{t \in \mathbb{N}} \) and policy \( \{ r_t (h^t) \}_{t \in \mathbb{N}} \) as fixed for each measurable \( h^t \), \( B_0 (h^0) \) and \( B_0^* (h^0) \) are known. Denote a measurable selection as \( X_t (h^t) = \{ X_t \} \). Define the real exchange rate as \( Q_t := S_t P_t^* / P_t \). Given the functional forms, the respective first order

\(^3\)Galí and Monacelli [2005] assume the existence of an international market for complete state-contingent claims. In doing so, they thus avoid the problem of steady-state allocations being dependent on initial conditions. McCallum and Nelson [1999] assume incomplete markets which would mean the opposite for steady state consumption; but this issue is not discussed by the authors.

\(^4\)Other ways of closing open-economy models are also discussed in Schmitt-Grohé and Uribe [2003]. In our framework the most natural alternative could be to assume endogenous transaction cost in taking position in foreign bonds (see, e.g., Benigno and Thoenissen [2008]). The model with this alternative assumption would be analytically less tractable, and the equilibrium dynamics requires a specific law of motion for bonds. Our assumption will make clear that what is crucial for the policy trade off is just the incompleteness of financial markets, and not the random walk property of the asset/consumption dynamics implied by this incompleteness (in the absence of the endogenous discounting assumption).
conditions of the household’s problem, for each $h^t$ and $t \in \mathbb{N}$, are:

\[
\psi N_i^h C_i^e = \frac{W_i}{P_i},\quad (2.4)
\]

\[
C_i^e = (1 + r_i) \mathbb{E}_t \left\{ \beta \left( C_i^h \right) \left( \frac{P_i}{P_{t+1}} \right) C_{i+1}^e \right\},\quad (2.5)
\]

\[
C_i^e = (1 + r_i) \mathbb{E}_t \left\{ \beta \left( C_i^h \right) \frac{P_i^t Q_{t+1}^h}{P_{t+1}^t Q_t} C_{i+1}^e \right\}.\quad (2.6)
\]

Each optimally chosen $C_t$ will be consistent with the household’s intra-period choice of a home-produced final consumption good, $C_{H,t}$ and an imported final good $C_{F,t}$, where $C_t$ is defined by a CES aggregator

\[
C_t = \left[ (1 - \gamma)^{\frac{1}{\gamma}} (C_{H,t}^\gamma + \gamma (C_{F,t}^{1-\gamma}) \right)^{\frac{\gamma}{1-\gamma}}; \quad \gamma \in (0,1), \eta > 1. \quad (2.7)
\]

Furthermore, each type of final good, $C_{H,t}$ and $C_{F,t}$, are aggregates of a variety of differentiated goods indexed by $i, j \in [0,1]$. Respectively, these aggregates are $C_{H,t} = \left[ \int_0^1 C_{H,t}(i) \frac{d\nu}{\nu} \right]^{\frac{1}{\nu}}$, and $C_{F,t} = \left[ \int_0^1 C_{F,t}(j) \frac{d\nu}{\nu} \right]^{\frac{1}{\nu}}$, where $\varepsilon > 1$. As is well known from Gali and Monacelli [2005], optimal allocation of the household expenditure across each good type gives rise to static demand functions for $(C_{H}(i), C_{F}(i), C_{H}, C_{F})$ and price indexes. Details of these demand functions and prices are given in a supplementary appendix.5

### 2.2 Differentiated goods technology and pricing

Each domestic firm $i \in [0,1]$ produces a differentiated good. Production is represented by a CES technology

\[
Y_t (i, h^t) = \left\{ \alpha \left[ A_i N_i^t (i, h^t) \right]^\nu + (1 - \alpha) \left[ IM_i (i, h^t) \right]^\nu \right\}^\frac{1}{\nu}; \quad \alpha \in (0,1], -\infty \leq \nu \leq 1, \quad (2.8)
\]

where $N_i(i)$ is labor hired by the firm and $IM_i(i)$ is an index of imported intermediate goods.6

The random variable $A_t := \exp\{a_t\}$ is an exogenous embodied labor productivity; $\alpha \in (0,1]$ measures the productive dependence of the domestic economy; and $1/(1-\nu)$ is the elasticity of substitution between labor and the imported intermediate good. The law of one price and the limiting closed-economy assumption for the rest of the world ensures that the price of $IM_t$ is equal to $S_t P_t^*$.

The production cost-minimization problem for the firm is:

\[
\min P_t (h^t) \left\{ Q_t (h^t) IM_t (i, h^t) + \frac{W_t (h^t)}{P_t (h^t)} N_i^t (i, h^t) \right\}.
\]

5See Supplementary Appendix A.

6This intermediate good can be interpreted as two equivalent forms. On the one hand, we can assume that the imported goods can be either devoted to consumption $C_{F,t}$ or used as a production input $IM_t$. On the other hand, we can assume that the domestic economy imports two differentiated goods: consumption $C_{F,t}$ and intermediated goods $IM_t$. 


With a homogeneous of degree one production function, the first-order conditions can be written in the aggregate as

$$\frac{W_t(h^t)}{P_t(h^t)} = \alpha \frac{MC^n_t(h^t)}{P_t(h^t)} A^n_t \left( \frac{Y_t(h^t)}{N^n_t(h^t)} \right)^{1-\nu}$$

(2.9)

and

$$Q_t(h^t) = (1 - \alpha) \frac{MC^n_t(h^t)}{P_t(h^t)} \left( \frac{Y_t(h^t)}{IM_t(h^t)} \right)^{1-\nu},$$

(2.10)

where \(MC^n_t\) is nominal marginal cost.

Since the firm \(i \in [0, 1]\) is assumed to be imperfectly competitive, it gets to set an optimal price \(P_{H,t}(i, h^t)\) given a Calvo-style random time-independent signal to do so. With a per-period probability \((1 - \theta)\) the firm gets to reset price. As this is quite a standard model in the literature, we derive the details separately (see Supplementary Appendix B). The firm’s optimal pricing decision is characterized by the first-order condition:

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k \left( \prod_{\tau=t}^{t+k-1} \beta(C^*_\tau) \right) \frac{\xi_t^i + k}{\xi_t} Y_{t+k}(i) \left[ \bar{P}_{H,t}(i) - \left( \frac{\varepsilon}{\varepsilon - 1} \right) MC^n_{t+k} \right] \right\} = 0,$$

(2.11)

where \(\xi_t := U_C(C_t)\), and the demand faced by the firm at some time \(t + k\) (and following history \(h^{t+k}\)), conditional on the firm maintaining a sale price of \(\bar{P}_{H,t}(i)\) is

$$Y_{t+k}(i) = \left( \frac{\bar{P}_{H,t}(i)}{\bar{P}_{H,t+k}} \right)^{-\varepsilon} [C_{H,t+k} + C^*_{H,t+k}].$$

(2.12)

In a symmetric pricing equilibrium, where \(\bar{P}_{H,t} := \bar{P}_{H,t}(h^t) = \bar{P}_{H,t}(i, h^t)\), the law of motion for the aggregate price is \(P_{H,t} = \left( \theta P_{H,t-1}^{1-\varepsilon} + (1 - \theta) \bar{P}_{H,t}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}\).

### 2.3 Market clearing

In a competitive equilibrium we require that given monetary policy and exogenous processes, the decisions of households and firms are optimal, as characterized earlier, and that markets clear. First, the labor market must clear, so that (2.4) equals (2.9) for all states and dates. Second, the final Home-produced goods market for each variety \(i \in [0, 1]\) clears so that:

$$Y_t(i, h^t) = C_{H,t}(i, h^t) + C^*_t(i, h^t).$$

(2.13)

Third, the no-arbitrage condition for international bonds will be given by the equality of (2.5) and (2.6). In the rest of the world, assumed to be the limiting case of a closed economy, we have market clearing as \(Y^*_t = C^*_t\).
3 Local equilibrium dynamics

In this section we characterize the log-linearized equilibrium dynamics of our small open-economy. To this end, consider the gaps of the aggregate variables with respect to their potential level in an equilibrium with fully flexible domestic prices—i.e., when the percentage deviation (from steady state) of real marginal cost, denoted by $mc_t$, is zero at any time $t$ and in any state. Let lowercase variables denote the percentage deviation of its level $X$ from its nonstochastic steady state point $X_{ss}$, e.g., $x := \ln(X/X_{ss})$. Define the potential output and the real exchange rate, respectively, $\bar{y}_t$ and $\bar{q}_t$, as the levels of output and real exchange rate, respectively, at the flexible-price equilibrium. It can be shown that the levels of both $\bar{y}_t$ and $\bar{q}_t$ only depend on exogenous variables. Let $\tilde{x}_t$ and $\tilde{q}_t$ denote the domestic output gap and the real exchange rate gap (in percentage deviation), respectively, where $\tilde{x}_t = y_t - \bar{y}_t$ and $\tilde{q}_t = q_t - \bar{q}_t$. It can be shown that the equilibrium dynamics can be fully approximated to first-order accuracy as a system of stochastic dynamic equations for $\tilde{x}_t$, $\pi_{H,t}$, and $\tilde{q}_t$. (Detailed derivations are provided in Section C of an appendix available separately.)

**Definition 1** Given a monetary policy process $\{r_t\}_{t \in \mathbb{N}}$ and exogenous processes $\{\epsilon_t, u_t\}_{t \in \mathbb{N}}$, a rational expectations competitive equilibrium is a bounded stochastic process $\{\pi_{H,t}, \tilde{x}_t, \tilde{q}_t\}_{t \in \mathbb{N}}$ satisfying:

\[
\pi_{H,t} = \hat{\beta}E_t \{\pi_{H,t+1}\} + \lambda (\kappa_1 \tilde{x}_t + \kappa_2 \tilde{q}_t),
\]

\[
\tilde{x}_t = \omega E_t \{\tilde{x}_{t+1}\} - \mu [r_t - E_t \{\pi_{H,t+1}\}] + \chi E_t \{\tilde{q}_{t+1}\} + \epsilon_t,
\]

\[
\tilde{q}_t = E_t \{\tilde{q}_{t+1}\} - (1 - \gamma) [r_t - E_t \{\pi_{H,t+1}\}] + u_t.
\]

where

\[
\lambda = \frac{(1 - \nu)(1 - \delta)}{1 - \nu + \delta \varphi} \left[\frac{(1 - \theta) (1 - \theta \hat{\beta})}{\theta}\right],
\]

\[
\kappa_1 = \varphi + \frac{\sigma}{1 - \gamma}, \quad \kappa_2 = \frac{\delta(1 - \nu + \varphi)}{(1 - \gamma)(1 - \nu)(1 - \delta)} - \frac{\sigma \eta \gamma (2 - \gamma)}{(1 - \gamma)^2} + \frac{\gamma}{(1 - \gamma)};
\]

\[
\omega = \frac{\sigma}{\sigma - \varphi}, \quad \mu = \left[\frac{1 - \gamma}{\varphi} \left[1 - \gamma + \eta \gamma \frac{(2 - \gamma)(\sigma - \varphi)}{1 - \gamma}\right] \right], \quad \chi = \eta \gamma \frac{\varphi (2 - \gamma)}{(1 - \gamma)(\sigma - \varphi)};
\]

\[
\delta = (1 - \kappa)(1M_{ss}/Y_{ss})^\nu, \text{ with } 1M_{ss} \text{ and } Y_{ss} \text{ as the stationary values of imported intermediate good and aggregate output, respectively.}
\]

3.1 Discussion: Irreducible equilibrium exchange rate dynamics

Consider the equilibrium IS functional equation (3.2). In our small open economy the real exchange rate indirectly affects the output gap via the ex-ante real interest rate (through $\mu$). This indirect channel is similar to the standard models of Gali and Monacelli [2005] and Clarida et al. [2001], and, depends on the degree of openness $\gamma$.\(^7\) Note however, movements in the

---

\(^7\)The depreciation in the real exchange rate raises the consumer price index because: (i) it increases the price of the imported consumption goods; and (ii) increases export demand, which results into a rise in the price of domestically produced goods. This increase in the consumer price index reduces the expected rate of inflation for a given expected future price level. This leads to an increase in the real interest rate facing consumers and,
conditional expectation of the real exchange rate in our model also affect the output gap (via $\chi$) directly: (i) by modifying the marginal rate of substitution of consumption between different periods and across states (i.e. $\phi$); and (ii) the interaction of these effects with the substitution between home and foreign-produced good (via $\eta$). This direct channel is just an artefact of the endogenous discount rate model, and, is negligible when we assume the limiting case for the elasticity of the discount rate with respect to aggregate consumption, $\phi \searrow 0$. In this case, $\chi \approx 0$. This assumption merely follows Ferrero et al. [2007].

Equation (3.1) is an augmented New Keynesian Phillips curve representing the dynamics of the short-run aggregate supply. In contrast to the standard model [e.g. Clarida et al., 2001; de Paoli, 2009a], we do not need to assume exogenous “cost-push shocks” in order to create a non-trivial monetary policy trade-off. The relevant monetary-policy trade-offs embedded in the Phillips curve, between $\tilde{\lambda}$ and $\pi_{H,t}$, is now driven by this endogenous “cost-push” channel (via $\lambda\kappa_2$). In our model, the direct link between real exchange rate movements and the real marginal cost can be dissected into two effects: production and demand channels. These further correspond to the three terms in the composite parameter $\kappa_2$ in equation (3.1). First, we have the channel on the production side (i.e. the first term in $\kappa_2$). Exchange rate depreciation in our model increases the unit price of the imported intermediate good, which drives the marginal cost and therefore domestic goods inflation up. Second, on the demand side, an increase in the real exchange rate (or an exchange rate depreciation) increases the prices of the imported consumption goods faced by domestic consumers. This effect has a substitution and an income effect on the marginal cost and the domestic inflation. On the one hand (i.e. the second term in $\kappa_2$), this increase in the price of imported consumption goods leads consumers to reduce the demand of these goods and therefore to reduce aggregate consumption and to increase leisure. This translates into an increase in marginal product of labor that drives the marginal cost up. On the other hand (i.e. the third term in $\kappa_2$), this increase in the price of imported consumption goods reduces the real wage rate facing by consumers, who react by increasing labor supply to compensate the reduction in the purchasing power of their given income. This leads to a reduction in marginal product of labor that pushes the marginal cost down. Observe that the substitution effect dominates, so that the net effect of the increase in the price of the imported consumption goods on marginal cost is always negative.

Therefore, the overall impact of the real exchange rate on the domestic inflation (i.e., the sign of $\kappa_2$) is then ambiguous, and it depends on the degree of openness $\gamma$ and the productive dependence $\alpha$ (through their effect on $\delta$). For empirically plausible parametrization, we show, in Section 4.1, that the overall sign of this slope $\lambda \kappa_2$ is negative.

Finally, note that our general model admits two other sources through which the exchange therefore, a decrease in consumption and output for a given expectation of the future exchange rate. Note however, the productive dependence given by $\delta$ does not affect the sensitivity of output gap to changes in real exchange rate because this parameter only determines the supply-side channel.


8Furthermore, $\phi$, affects the elasticities of output gap with respect to the ex-ante real interest rate $\mu$. Again, with $\phi \searrow 0$, this indirect channel introduced by endogenous discounting will be negligible.

9See Clarida et al. [1999] for a detailed discussion on this ad-hoc cost-push term.

10In our model the real marginal cost is not fully tied to the output gap but also depends on the real exchange rate as is shown in Section C of the supplementary appendix. Moreover, as (3.3) shows, the dynamics of the real exchange rate depends on the exogenous variable $u_t$ for an the endogenous nominal interest rate $r_t$ policy outcome. This feature of our model does not rely on price stickiness in an additional imported goods sector as in Monacelli [2005].
rate may explicitly matter: (i) the endogenous discount factor channel, which as we have just discussed, remains inconsequential to this result (i.e. when \( \phi \downarrow 0 \)); and (ii) the possibility of an imported input in the small economy’s production structure. Again, this latter channel remains inconsequential to the existence of an irreducible equilibrium exchange rate dynamics. To see this, one can just take the zero limit of the share of the imported input in the production technology \((1 - \alpha) \downarrow 0\) (i.e. \( \delta \downarrow 0 \)). Even in this case the term \( \kappa_2 \neq 0 \), implying that the exchange rate dynamics is still irreducible and is still present as an endogenous-variable “cost-push” term.\(^{11}\) We have thus shown that in the limit when these two sources are removed, an irreducible exchange rate dynamics still remains; and this is purely a result from the existence of incomplete international asset markets. We next discuss what this feature does to the model.

### 3.2 Role of incomplete markets in the equilibrium relations

What is the role of incomplete international asset markets in the equilibrium characterization in our model? To answer this question, we derive the complete-markets limit of our model, which is similar in reduced-form, but slightly different in detail, to Gali and Monacelli [2005], because of our assumption on the openness in the production side (i.e., \( \delta \neq 0 \)). In the complete markets version of our model, complete international risk sharing results in a tight link between the real exchange rate and the marginal rate of substitution of cross-country consumption, \( q_t = \sigma (c_t - c^*_t) \), in every state of the nature.\(^{12}\) Using this relationship we obtain that

\[
y_t = \left[ \frac{(1 - \gamma)^2 + \sigma \eta \gamma (2 - \gamma)}{\sigma (1 - \gamma)} \right] q_t + y^*_t. \tag{3.4}
\]

This implies that output gap is a linear function of the real exchange rate gap:

\[
\tilde{x}_t = \left[ \frac{(1 - \gamma)^2 + \sigma \eta \gamma (2 - \gamma)}{\sigma (1 - \gamma)} \right] \tilde{q}_t.
\]

By using this resulting complete-markets relation, we can deduce the following differences between our model and the limit economy with complete markets. First, in the complete-markets version the exchange rate does not affect domestic inflation explicitly, i.e., \( \kappa_2 \equiv \kappa^c_2 = 0 \).\(^{13}\) This implies that the complete-markets version of our model implies an isomorphic monetary policy design to that of the closed-economy model, as in Clarida et al. [2001]. Thus \( \kappa_2 \neq 0 \) implies that incompleteness in international asset markets introduces endogenously a cost push in our Phillips curve. This observation is still valid as long as we assume that the effect of endogenous discounting is negligibly small: \( \phi \downarrow 0 \).

Second, international asset market incompleteness also affects the response of inflation to the output gap. Effectively, the parameter \( \kappa_1 \) is different in the complete-markets version. In

\(^{11}\)This “irreducibility” of the real exchange rate in equilibrium is not true, even in the general parametric case of the complete markets models as in Benigno and Benigno [2003]. This observation also extends to the complete-markets small open economy used in the literature.

\(^{12}\)With complete markets, the Euler condition (within the conditional expectations operator) in (2.5) will in fact hold for every state of nature, following every history, such that equating the Home Euler condition to that of the rest of the world, one can derive the condition that \( Q_t = (C^*_t / C_t)^{-\sigma} \), and a log-linear transform of this expression is \( \tilde{q}_t = \sigma (c_t - c^*_t) \).

\(^{13}\)We will use the super-script \( c \) to denote the parameters that are different in the complete market version of the model.
particular, it would be

$$\kappa_1^c = \left\{ \frac{1 + \delta (\varphi - \nu)}{(1 - \delta) \left[ 1 - \gamma + \frac{\gamma \eta(2 - \gamma)}{1 - \gamma} \right]} \right\} \left( \frac{\sigma}{1 - \gamma} \right) + \varphi,$$

in the limit of our economy with complete markets. The difference between \(\kappa_1\) (incomplete markets) and \(\kappa_1^c\) (complete markets) is the first expression in the right-hand side of \(\kappa_1^c\). This difference has an ambiguous sign because a priori it is difficult to sign \(\varphi - \nu\). However, we will be considering negative values of \(\nu\), following McCallum and Nelson [1999]. Hence, this first expression in the right-hand side of \(\kappa_1^c\) will be positive. However, we do not yet know whether this expression is larger or smaller than one. In Section 5 we will quantify this difference.

Third, market incompleteness also affects the response of output gap to the real interest rate given by \(\mu\). In the complete market version this parameter would be

$$\mu^c = \left[ \frac{1 - \gamma}{\sigma - \phi (1 - \gamma)} \right] \left[ 1 - \gamma + \frac{\gamma \eta (2 - \gamma)}{1 - \gamma} \right].$$

However, provided that \(\phi\) takes values very close to zero, then \(\mu \approx \mu^c\). Therefore, the effect of market incompleteness in the response of output gap to real interest rate will be negligible.

**Proposition 1** If the small open economy has access to complete international securities markets, then the real exchange rate is reducible from the equilibrium characterization—i.e. has no fundamental role in the dynamic characterization of equilibrium. The competitive equilibrium is then characterized by

$$\pi_{H,t} = \beta \mathbb{E}_t \{ \pi_{H,t+1} \} + \lambda \kappa_1^c \bar{x}_t,$$

$$\bar{x}_t = \omega \mathbb{E}_t \{ \bar{x}_{t+1} \} - \mu^c \left[ r_t - \mathbb{E}_t \{ \pi_{H,t+1} \} \right] + \epsilon_t. \quad (3.5)$$

Observe that these direct effects of the real exchange rate in the dynamics of the domestic inflation \(3.1) through marginal cost also disappear either when \(\delta = \gamma = 0\). Furthermore, if \(\phi \approx 0\), then there is no direct real exchange rate channel in the IS relation \(3.2) as well.

**Proposition 2** If the economy: (i) does not rely on imported intermediate inputs and final consumption goods, \(\delta = \gamma = 0\), and (ii) thus endogenous discounting is an irrelevant assumption, so that without loss of generality, \(\phi \approx 0\), then, the model is equivalent to the canonical new-Keynesian closed-economy model. This limit economy is isomorphic to the complete-markets small open economy characterized by \(3.5) and \(3.6).

### 3.3 Endogenous monetary policy trade-off

We begin with a simple observation that the model no longer inherits an isomorphic monetary policy design problem to its closed economy counterpart. In the canonical model of Gali and Monacelli [2005], an equivalent exogenous variation in \(\epsilon_t\) can be fully offset by the monetary policy instrument, \(r_t\), thus removing any impact of exogenous shocks on domestic output gap \(\bar{x}_t\). This is the reason for the non-existence of any policy trade-off between domestic producer price inflation \(\pi_{H,t}\) and output gap \(\bar{x}_t\). This is also the case, qualitatively, in the closed economy canonical model [see e.g. Woodford, 2003] and in our nested cases when \(\delta = \gamma = 0\) and \(\phi = 0\).
Typically, to ensure that there is a relevant monetary policy trade-off, the literature using the canonical model would introduce exogenous “cost-push” shocks [see e.g. Woodford, 2003] to the special case of the Phillips equation (3.1).

In our model, bond market incompleteness results in the equilibrium relation (3.3) and also the explicit linkage between the real exchange rate and real marginal cost, and therefore, the domestic producer price inflation in (3.1). The resulting monetary policy implication is that any exogenous variation encapsulated in $\varepsilon_t$ (i.e. foreign output, foreign real interest rate, or domestic labor productivity) cannot be fully offset by changing $r_t$, since this will also affect the real exchange rate via (3.3) and domestic inflation via (3.1).

However, recall that general parametric settings of the open economy with complete markets [see e.g. Benigno and Benigno, 2003] may also yield monetary policy that is also no longer isomorphic to its closed economy limit. However, in contrast to these results on policy non-isomorphism, ours is driven by an explicit and irreducible exchange rate channel.

4 Parametrization

Our baseline economy (hereinafter also referred to as “Open IM”) is parametrized in line with Llosa and Tuesta [2008] and McCallum and Nelson [1999]. Llosa and Tuesta [2008] use the same calibration as Galí and Monacelli [2005] with the exception of the constant relative risk aversion coefficient ($\sigma$), the inverse of Frisch labor supply elasticity ($\varphi$), and the elasticity of substitution between domestic and foreign goods ($\eta$). For a majority of parameters, we follow that of in Llosa and Tuesta [2008] for two reasons: (i) Ease of comparison of their findings with ours in terms of the REE stability analyses; and (ii) The settings in Llosa and Tuesta [2008] is a more general parametrization. Furthermore, these parameters does not affect qualitatively the results, although they may have important quantitative effects. This is mainly true in the case of $\sigma$. The parameter $\nu$ is from McCallum and Nelson [1999]. We summarize the model parameters in Table 1.

[ Table 1 about here. ]

Note that this set of parameters is later used to consider the limits of our model. That is, by using the relevant composite parameters, we have: (i) The small open economy with complete markets (“Open CM”): $\kappa_2 = 0$, $\kappa_1 = \kappa_1^c$ and $\mu = \mu^c$; and, (ii) The closed economy (“Closed”): $\gamma = 0$ and $\alpha = 1$ (i.e., $\delta = 0$).

4.1 Numerical values of equilibrium relations

By using our baseline parametrization we can now sign the reduced-form parameters in the equilibrium relations between variables in the approximation of the competitive equilibrium (3.1), (3.2) and (3.3). We report this in Table 2.

---

[14]The goal in this paper is to understand the qualitative implications of incomplete markets on equilibrium stability using a simple but salient model, and not to quantify or match business cycle regularities. However, we do perform some sensitivity analysis in this parameter when it would be required. Results of these alternative experiments will be readily made available upon request.
Recall that the only unclear equilibrium relation (in terms of sign or direction) is the slope of output gap with respect to the real exchange rate in the Phillips curve (3.1), given by $\lambda \kappa_2$. From a Mundell-Fleming-type intuition, one should expect that without imported inputs this value should be negative. We show that in our baseline economy, this value is also negative with imported inputs. In fact, we obtain that $\kappa_2$ is positive if and only if $v \in (0.92337, 1)$. Therefore, the aforementioned income effects of the exchange rate variations dominates.

At this point, it is convenient to enumerate the differences between the incomplete market version and the complete market version in terms of equilibrium relations. Remember that these differences are in the relations $\lambda \kappa_1$, $\lambda \kappa_2$ and $\mu$. Table 3 reports the differences across the economies with and without openness in production, and with and without market incompleteness.

From Table 3 we conclude the following. First, the positive response of the inflation rate to the output gap, given by $\lambda \kappa_1$, is much larger (i.e. around six times larger) with incomplete markets, regardless of the presence of an imported intermediate input channel (indexed by $\delta$). The intuition is quite standard. In the absence of complete international risk sharing, a given external shock to the small open economy cannot be fully insured against by the single incomplete market claim. Hence the effect of the shock gets amplified or transmitted more to domestic allocations via the inflation process.

Second, the relationship between domestic-goods inflation rate and output gap, given by $\lambda \kappa_1$, is decreasing with $\delta$ (openness in production). Observe the Phillips curve relation in (3.1). Since there is some substitutability in the inputs in the production technology, more reliance (i.e. higher $\delta$) on the imported intermediate input means a substitution away from the labor input, such that all else equal, the effect of the labor cost channel on the inflation-real marginal-cost channel via $\lambda \kappa_1$ is then smaller.

Third, the relationship between domestic-goods inflation rate and output gap, given by $\lambda \kappa_1$, in the closed economy (“Closed”) is between the value in the incomplete market version and the complete market version.

Fourth, the openness in production, given by $\delta$, reduces the negative effect of the depreciation of the exchange rate on the inflation rate, given by $\lambda \kappa_2$. This is obvious because with $\delta = 0$, the production effect determining $\kappa_2$ disappears.

Fifth, given that $\phi$ is very close to zero, the response of the output gap to the interest rate, given by $\mu$ is the same in the two versions of open economies. Last, the response of the output gap to the interest rate, given by $\mu$ is much smaller in the closed economy. The reason for this is as in Gali and Monacelli [2005] – viz. trade openness presents an indirect terms of trade (or real exchange rate) variation on aggregate demand.

5 Implications for Instrument Rules and Determinacy

In this section we show how the additional incomplete-asset-markets friction alters the space of alternative policy rules that can feasibly deliver a unique REE. Incomplete asset markets
result in drastically different implications, in contrast to well-received wisdom from the closed-economy [e.g. Bullard and Mitra, 2002] and complete markets small-open-economy [e.g. Llosa and Tuesta, 2008], for REE stability under various policy rules. We will consider six classes of simple contemporaneous and forecast-based Taylor-type monetary policy rules used in the literature [see e.g. Llosa and Tuesta, 2008; Bullard and Mitra, 2002].

(i) the domestic (producer price) inflation targeting rule (DITR):

\[ r_t = \phi_\pi \pi_{H,t} + \phi_x \tilde{x}_t; \] (5.1)

(ii) a forecast-based version of the DITR (or FB-DITR):

\[ r_t = \phi_\pi E_t \{ \pi_{H,t+1} \} + \phi_x E_t \{ \tilde{x}_{t+1} \}; \] (5.2)

(iii) the consumer price index inflation targeting rule (CPITR):

\[ r_t = \phi_\pi \pi_t + \phi_x \tilde{x}_t; \] (5.3)

(iv) a forecast-based CPITR (or FB-CPITR):

\[ r_t = \phi_\pi E_t \{ \pi_{t+1} \} + \phi_x E_t \{ \tilde{x}_{t+1} \}; \] (5.4)

(v) the managed exchange rate targeting rule (MERTR):

\[ r_t = \phi_\pi \pi_{H,t} + \phi_x \tilde{x}_t + \phi_s \triangle s_t; \] (5.5)

(vi) a forecast-based MERTR (FB-MERTR):

\[ r_t = \phi_\pi E_t \{ \pi_{H,t+1} \} + \phi_x E_t \{ \tilde{x}_{t+1} \} + \phi_s E_t \{ \triangle s_{t+1} \}, \] (5.6)

where the elasticities \( \phi_\pi, \phi_x, \) and, \( \phi_s \) are non-negative reaction parameters.

Where relevant, we will compare within each policy class, the REE stability and indeterminacy implications across the three economies: (a) The small open economy with complete markets (“Open CM”) limit; (b) the closed economy limit (“Closed”); and (c) the encompassing small open economy with incomplete markets model (“Open IM”).

Given each policy rule above, and the competitive equilibrium conditions (3.1), (3.2) and (3.3), the equilibrium system can be reduced to

\[ E_t x_{t+1} = A x_t + C w_t, \] (5.7)

where \( x := (\pi_{H}, \tilde{x}, \tilde{q}) \), and \( w := (\varepsilon, u) \), and \( A := A(\bar{\theta}, \bar{\phi}) \) and \( C := C(\bar{\theta}, \bar{\phi}) \) depend on the parameters in (3.1), (3.2) and (3.3), \( \bar{\theta} := (\bar{\beta}, \gamma, \kappa_1, \kappa_2, \omega, \mu, \chi) \); and also the policy parameters \( \bar{\phi} := (\phi_\pi, \phi_x, \phi_s) \).\(^{15}\)

\(^{15}\)More details are available in a separate appendix as Section D.1.
Local stability of a REE depends on the eigenvalues of matrix $A$. Following the terminology of Blanchard and Kahn [1980], we can see that there are three non-predetermined variables. Therefore, the equilibrium under DITR will be determinate if the three eigenvalues of $A$ are outside the unit circle, whereas it will be indeterminate when at least one of the three eigenvalues of $A$ is inside the unit circle. Unfortunately, we are not able to obtain analytical characterizations of the stability conditions for each class of policy rules. We numerically check for determinate REE (and similarly check for multiplicity of REE) as functions of the policy parameters. In particular, we consider $\phi_\pi \in [0, 4]$ and $\phi_x \in [0, 4]$, as in Llosa and Tuesta [2008], and vary $\phi_s$ where relevant.

What we have shown previously, is that the inability of a small open economy to completely insure its country-specific technology risk results in an endogenous trade-off for monetary policy. This outcome does not exist in the canonical NK closed economy nor the complete-markets small open economy models (recall the discussion in section 3.1). This feature poses additional restrictions on the admissibility of the classes of policy rules in terms of inducing a determinate REE.

We will illustrate the following conclusions. First, market incompleteness results in an opposite conclusion to the finding in Llosa and Tuesta [2008]. Llosa and Tuesta [2008] showed that the set of admissible DITR (that respond to contemporaneous variables) inducing unique REE, in a small open economy with complete markets, is larger than that in its closed-economy limit. In general, we find that market incompleteness makes the admissible policy sets smaller than when we have the limit of the complete-markets small open economy. In the specific case of the DITR, international asset market incompleteness also reduces the admissible policy space relative to when we have the closed-economy limit. Second, if the policy rules are of the forecast-based families (FB-DITR, FB-CPITR and FB-MERTR), then market incompleteness in our model also shrinks the sets of these policies that can induce unique REE, relative to their counterparts in the special case of the complete-markets small open economy model. Third, if monetary policy can be described by simple policy rules, then a contemporaneous rule (MERTR) that not only responds to domestic inflation and output gap, but also to the real exchange rate growth, can greatly expand the feasible set of such policies in inducing determinate rational expectations equilibrium. This result is also well-known in the context of small open economies with complete markets [see e.g. Llosa and Tuesta, 2008].

**Result 1** Consider the monetary policy rules DITR, CPITR, MERTR, FB-DITR, FB-CPITR and FB-MERTR. Then,

(i) In each of these rules, the relevant set of policy parameters $(\phi_x, \phi_\pi)$ that induce REE indeterminacy is larger in the open economy with incomplete international risk sharing (Open IM) than in the open economy with complete markets (Open CM).

(ii) In the case of DITR and CPITR, the set of policy parameters $(\phi_x, \phi_\pi)$ that induce REE indeterminacy in the closed economy limit is larger than that of the Open CM limit.
5.1 Illustration of numerical results

DITR. Figure 1 reports the simulation results for DITR (5.1) across the three economies: (i) the small open economy with complete markets (“Open CM”) limit; (ii) the closed economy limit (“Closed”); and (iii) the encompassing small open economy with incomplete markets model (“Open IM”). Each shaded region refers to the set of DITR policy rules, indexed by \((\phi_\pi, \phi_x)\), that would have induced a determinate REE in each of the economies Open CM, Closed, and Open IM. The complement set of each shaded region represents the region with multiple or indeterminate REE.

We observe that the largest value of \(\phi_\pi\) for which indeterminacy arises is 1 which corresponds with \(\phi_x = 0\). The largest value of \(\phi_x\) for which we find indeterminacy is 4, which corresponds to \(\phi_\pi = 0.96\). In fact, this point \((\phi_\pi, \phi_x) = (1, 0.96)\) determines the length of indeterminacy, and therefore, the constraint faced by the policy makers in setting a DITR as a stabilizing device. In particular, the monetary authority is not constrained if the policy reaction to inflation \(\phi_\pi\) is larger than unity (i.e. the “Taylor principle”). However, provided that \(\phi_\pi < 1\), the smaller this policy parameter is, the greater the authority’s responses to the output gap.

We note in the following, a qualification of existing results [e.g. Llosa and Tuesta, 2008] that openness to trade reduces indeterminacy of REE under the DITR family of policy rules. Now, openness to trade under complete markets (Open CM) reduces the set of DITR policy that induces REE indeterminacy, compared to the Closed economy. However, incomplete asset markets (Open IM) expands the set of indeterminate REE from that of the Closed economy. In other words, while trade openness reduces the constraint for DITR policy makers if markets are complete, this openness increases the constraints if markets are incomplete.\(^{16}\) The intuition for this is not surprising. As discussed in Sections 3.2 and 3.3, and also in Table 3, market incompleteness exacerbates the slope of the inflation-output-gap trade-off and further introduces feedback on inflation via the real exchange rate. The additional sensitivity of inflation to output gap and the real exchange rate in the Phillips curve means that a DITR policy maker in the Open IM economy will have to counter movements in inflation much more than its counterparts in the Open CM or Closed economies, to deliver a determinate REE.

FB-DITR. Figure 2 shows the sets of FB-DITR policies (5.2) that induce unique REE for our Open IM economy, the limit economy Open CM and also the limit of the Closed economy. Again, we see that the set of FB-DITR inducing indeterminate REE is larger than that of DITR, and the set of rules inducing indeterminate REE is larger in the Open IM economy than in the Open CM economy. This set is larger in the Open CM economy than in the Closed economy.

\(^{16}\)We also check whether this result holds when the risk aversion coefficient is lowered to that used in Gali and Monacelli [2005]. We can show that if we move from \(\sigma = 5\) of Llosa and Tuesta [2008] to \(\sigma = 1\) of Gali and Monacelli [2005], the qualitative differences in the determinate REE sets across the three economies become even more prominent. These results are available separately from the authors.
CPI TR. Next we also consider a variation on the DITR, the CPI Inflation Targeting Rule (CPI TR), as in equation (5.3). However, the authority and agents in the economy know that in equilibrium, there is a relationship between CPI, domestic inflation, and the real exchange rate, given by \( \pi_t = \pi_{H,t} + \frac{\gamma}{1 - \gamma} (\bar{q}_t - \bar{q}_{t-1}). \) Then the CPI TR can be equivalently written as

\[
 r_t = \phi_\pi \pi_{H,t} + \phi_x \bar{x}_t + \frac{\gamma \phi_\pi}{1 - \gamma} (\bar{q}_t - \bar{q}_{t-1}). 
\] (5.8)

Note that for the Open CM economy, we can use the complete cross-country risk sharing condition in (3.4), so that CPI TR in this limit economy becomes,

\[
 r_t = \phi_\pi \pi_{H,t} + \left( \phi_x + \frac{\tau \gamma \phi_\pi}{1 - \gamma} \right) \bar{x}_t - \frac{\tau \gamma \phi_\pi}{1 - \gamma} \bar{x}_{t-1}, \quad \tau := \frac{\sigma (1 - \gamma)}{(1 - \gamma)^2 + \sigma \eta (2 - \gamma)}. 
\] (5.9)

In the Closed economy case, CPI TR is identical to its DITR. As before, by combining (5.8), or (5.9), or the DITR with the equilibrium characterizations in each of the respective three economies, we can then numerically characterize the stability conditions for a REE under CPI TR.

Figure 3 depicts our findings for these rules. Openness to trade under complete markets (Open CM) reduces the set of CPI TR policy that induces REE indeterminacy, compared to the Closed economy. However, incomplete asset markets (Open IM) expands the set of CPI TR policy that induces indeterminate REE from that of the Open CM economy. Observe that qualitatively, the order of the indeterminacy sets under CPI TR are identical to DITR. However, we next show that this is no longer true if the authority follows a forecast based rule.

[ Figure 3 about here. ]

FB-CPI TR. Consider next, a forecast-based version of the CPI TR or FB-CPI TR in equation (5.4). Again, using the CPI inflation relation with domestic inflation and real exchange rate gap, this rule can be written as

\[
 r_t = \phi_\pi \mathbb{E}_t \{ \pi_{H,t+1} \} + \phi_x \mathbb{E}_t \{ \bar{x}_{t+1} \} + \frac{\gamma \phi_\pi}{1 - \gamma} (\mathbb{E}_t \bar{q}_{t+1} - \bar{q}_t), 
\] (5.10)

for the baseline Open IM economy, and,

\[
 r_t = \phi_\pi \mathbb{E}_t \{ \pi_{H,t+1} \} + \left( \phi_x + \frac{\tau \gamma \phi_\pi}{1 - \gamma} \right) \mathbb{E}_t \{ \bar{x}_{t+1} \} - \left( \frac{\tau \gamma \phi_\pi}{1 - \gamma} \right) \bar{q}_t, 
\] (5.11)

in the case of the Open CM economy.

Compared to the Closed economy, the set of FB-CPI TR inducing determinate REE is smaller. However, this set (Open CM) is larger than in the (Open IM) economy with FB-CPI TR.

[ Figure 4 about here. ]

MER TR. Consider the Managed Exchange Rate Taylor Rule (MER TR) in equation (5.5). Using the definition of the nominal exchange rate, \( \Delta s_t = \Delta q_t + \pi_t - \pi_t^*, \) where without loss of
generality we set $\pi^*_t = 0$, we then obtain the relation that $\Delta s_t = \frac{1}{1-\gamma} \Delta q_t + \pi_{H,t}$. Using this, we have an equivalent representation of the MERTR as

$$r_t = (\phi_\pi + \phi_s) \pi_{H,t} + \phi_x \tilde{x}_t + \left( \frac{\gamma \phi_\pi + \phi_s}{1-\gamma} \right) (\tilde{q}_t - \tilde{q}_{t-1}), \quad (5.12)$$

By combining this rule with the equilibrium conditions for the Open IM economy, we can again characterize REE stability numerically.

Similarly, we can derive a representation of the MERTR for the case of the Open CM economy as

$$r_t = (\phi_\pi + \phi_s) \pi_{H,t} + \left( \phi_x + \tau \left( \frac{\gamma \phi_\pi + \phi_s}{1-\gamma} \right) \right) \tilde{x}_t - \tau \left( \frac{\gamma \phi_\pi + \phi_s}{1-\gamma} \right) \tilde{x}_{t-1}, \quad (5.13)$$

where $\tau := \frac{\sigma (1-\gamma)}{(1-\gamma)^2 + \sigma \eta \gamma (2-\gamma)}$.

We fix $\phi_s = 0.6$ as in Llosa and Tuesta [2008]. Relative to the DITR, the admissible set of the MERTR inducing determinate REE equilibrium is larger. However, relative to Open CM economy, asset market incompleteness in the Open IM economy reduces this set.

[ Figure 5 about here. ]

**FB-MERTR.** It can be shown that the FB-MERTR rule, in equation (5.6) has an equivalent form of

$$r_t = (\phi_\pi + \phi_s) \pi_{H,t} + \left( \phi_x + \frac{\tau (\gamma \phi_\pi + \phi_s)}{1-\gamma} \right) \tilde{x}_t + \left( \frac{\gamma \phi_\pi + \phi_s}{1-\gamma} \right) \tilde{x}_{t-1}, \quad (5.14)$$

in the Open IM economy, and,

$$r_t = (\phi_\pi + \phi_s) \pi_{H,t} + \left[ \phi_x + \tau \left( \frac{\gamma \phi_\pi + \phi_s}{1-\gamma} \right) \right] \tilde{x}_{t+1} - \tau \left( \frac{\gamma \phi_\pi + \phi_s}{1-\gamma} \right) \tilde{x}_t, \quad (5.15)$$

in the Open CM economy, where $\tau := \frac{\sigma (1-\gamma)}{(1-\gamma)^2 + \sigma \eta \gamma (2-\gamma)}$. Relative to Open CM economy, asset market incompleteness in the Open IM economy results in a smaller admissible set of the FB-MERTR inducing determinate REE equilibrium.

[ Figure 6 about here. ]

**Discussion.** We have seen in the illustrations above that the existence of incomplete international risk sharing results in a reduction of the sets of admissible rules inducing determinate REE, relative to when the environment is the standard complete-markets small open economy. However, given international asset market incompleteness, the admissible set of policy rules can be expanded by a family of simple policies that take into account real exchange rate growth as well. This makes intuitive sense since the additional constraint on policy, or the endogenous

---

17 Additional sensitivity results with respect to $\phi_s$ are available from the authors. We show that the qualitative ordering of the sets of determinate or indeterminate equilibria are not affected by the feasible choice of this parameter.
policy trade-off, was a result itself of an irreducible and explicit real exchange rate dynamic channel.

6 Concluding remarks

In this paper, we have developed a small open economy whose monetary policy implications are no longer similar to its closed-economy counterpart. We showed, in a transparent manner, that asset market incompleteness essentially exposes the supply side of the model’s equilibrium characterization to a notion of an endogenous cost-push shock. Our notion of an endogenous cost-push trade-off here is different to existing models with complete markets. In our model, this is a consequence of an irreducible and explicit exchange rate equilibrium dynamic channel. Finally, we re-visit the lessons on equilibrium determinacy under alternative rules in a small open economy. We show that asset market incompleteness now results in opposite conclusions to the existing literature utilizing the workhorse complete markets model.

While our model is a simple and transparent illustration of the relation between international asset market incompleteness, equilibrium exchange rate irreducibility, and its implications for rational expectations equilibrium determinacy, it is probably too simple for normative business-cycle and welfare analysis. Larger and more quantitative models are more suited to addressing these normative questions [see e.g. de Paoli, 2009b; Monacelli, 2005].

References


Table 1: Parametrization for encompassing Open IM model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion, $\sigma$</td>
<td>5</td>
<td>LT</td>
</tr>
<tr>
<td>Disutility of labor, $\psi$</td>
<td>1</td>
<td>GM</td>
</tr>
<tr>
<td>Inverse Frisch elasticity, $\varphi$</td>
<td>0.47</td>
<td>LT</td>
</tr>
<tr>
<td>Discount factor elasticity, $\phi$</td>
<td>$10^{-6}$</td>
<td>LT</td>
</tr>
<tr>
<td>Steady state discount factor, $\beta$</td>
<td>0.99</td>
<td>GM</td>
</tr>
<tr>
<td>Home-Foreign goods elasticity of substitution, $\eta$</td>
<td>1.5</td>
<td>LT</td>
</tr>
<tr>
<td>Share of Home goods in $C$, $\gamma$</td>
<td>0.4</td>
<td>GM</td>
</tr>
<tr>
<td>Elasticity of substitution between good varieties, $\varepsilon$</td>
<td>6</td>
<td>GM</td>
</tr>
<tr>
<td>Price stickiness probability, $\theta$</td>
<td>0.75</td>
<td>GM</td>
</tr>
<tr>
<td>Labor-imported-input elasticity of substitution, $v$</td>
<td>$-2$</td>
<td>MN</td>
</tr>
<tr>
<td>Steady-state imported-input share of output, $\delta$</td>
<td>0.144</td>
<td>MN</td>
</tr>
</tbody>
</table>

Notes:
(a) GM: Galí and Monacelli [2005]
(b) LT: Llosa and Tuesta [2008]
(c) MN: McCallum and Nelson [1999]

Table 2: Baseline model’s RCE characterization

<table>
<thead>
<tr>
<th>Composite Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.0719</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>8.8033</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>$-12.3424$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>1.0000</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.0320</td>
</tr>
<tr>
<td>$\chi$</td>
<td>$3.2 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

Table 3: Comparing RCE characterizations

<table>
<thead>
<tr>
<th></th>
<th>Open IM</th>
<th>Open CM</th>
<th>Closed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta = 0.114$</td>
<td>$\lambda \kappa_1$</td>
<td>0.6325</td>
<td>0.7356</td>
</tr>
<tr>
<td>$\delta = 0$</td>
<td>$\lambda \kappa_2$</td>
<td>$-0.8868$</td>
<td>$-1.0872$</td>
</tr>
<tr>
<td>$\delta = \gamma = 0$</td>
<td>$\mu$</td>
<td>1.0320</td>
<td>1.0320</td>
</tr>
</tbody>
</table>
Figure 1: Domestic inflation targeting rule (DITR) and indeterminacy for three economies.
Figure 2: Forecast-based domestic inflation targeting rule (FB-DITR) and indeterminacy for three economies.
Figure 3: CPI inflation targeting rule (CPITR) and indeterminacy for three economies.
Figure 4: Forecast-based CPI inflation targeting rule (FB-CPTTR) and indeterminacy for three economies.
Figure 5: Managed exchange rate targeting rule (MERTR) and indeterminacy for three economies.
Figure 6: Forecast-based managed exchange rate targeting rule (FB-MERTR) and indeterminacy for three economies.