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## Tax-versus-trading and efficient revenue recycling as issues for greenhouse gas abatement

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### ABSTRACT

We give empirical welfare results for global greenhouse gas emission abatement, using the first multi-party model to include both tax-versus-trading under uncertainties, and revenue recycling. Including multiple, independent parties greatly reduces the welfare advantage of an emissions tax over emissions (permit) trading in handling abatement-cost uncertainties, from that shown by existing, single-party literature. But a previously ignored and much bigger advantage of a tax, from better handling uncertainties in business-as-usual emissions, greatly boosts the overall tax-versus-trading advantage. Yet the degree to which each mechanism is used to raise and recycle revenue efficiently by lowering distortionary taxes – rather than recycle revenue as lump sums, or not raise revenue by giving tax thresholds or free permits – may in turn dominate any tax-versus-trading advantage. Choosing the best greenhouse abatement mechanism should thus consider the issues of tax-versus-trading and efficient revenue recycling together.

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## 1. Introduction

Designing policy mechanisms for abating greenhouse gas emissions cost-effectively grows ever more important, as scientists recommend ever stronger abatement to avoid dangerous climate change [1]. For decades economists have promoted emission pricing – market mechanisms (economic instruments) like a carbon (emissions) tax or (carbon) emissions (permit) trading – for such abatement. Compared to directly regulating millions of greenhouse emitters, pricing can minimise total abatement costs by equalising emitters' otherwise very diverse marginal abatement costs. It can also avoid huge administration costs.

Also for decades, economists have debated which emission pricing mechanism is most cost-effective, especially the choice between direct “prices” (a tax) and indirect prices via “quantities” (tradable permits) under abatement-cost uncertainty, following Weitzman's [20] seminal, partial-equilibrium analysis. However, uncertainties other than in abatement costs also affect this choice, and we will show that uncertainties in each party's (country's or region's) future, business-as-usual (BAU) emissions are even more important. And since the early 1990s, another key issue in the mechanism-choice debate applied to greenhouse abatement has been the welfare-reducing, general-equilibrium interaction between emission pricing and existing, conventional taxes on other factors of production (labour and/or capital). This shows the importance of both raising revenue from emission pricing, and then recycling it to raise welfare—usually

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assumed to be done by cutting rates of factor taxation and thus the distortionary welfare losses of such taxation [3,4], even though this rarely happens in practice.

Our model is the first multi-party, theoretical and empirical model of greenhouse gas abatement to combine tax-versus-trading (with uncertainties in BAU emissions as well as in abatement costs, all assumed independent of each other, which turns out to be important) and an approximate, mainly partial-equilibrium treatment of revenue recycling. Quirion [16] is the only other combination of tax-versus-trading and (partial-equilibrium) revenue recycling, and his model was only theoretical and for one party. Our inclusion of many parties (firms or countries) is rare, perhaps because Weitzman's prices-versus-quantities welfare formula was for a single party. But any real emissions-pricing scheme does contain many parties, with diverse abatement costs.

We proceed by adding a tax, and an approximate revenue-recycling formula (assuming for the sake of reference that factor tax cuts are used), to the multi-party model of Mechanisms to Abate Total Emissions under Stochasticity (MATES) in Jotzo and Pezzey [7]. We compute the tax-versus-trading and revenue-recycling welfare advantages<sup>1</sup>; the latter rises if either mechanism's "efficient recycling share" (the share of emissions not exempted by tax thresholds or free permits) rises. Hybrid mechanisms, such as emissions trading with a price cap [15] or long-term permits with a short-run maximum price [9], could be important extra practical options, but the main issues here would still be relevant. Our empirical context is an 18-region world in 2020, representing a short run when the global greenhouse gas stock is very large compared to emissions flow.

## 2. A model with tax-versus-trading and efficient revenue recycling

### 2.1. The theoretical model

Emissions are perfectly mixed in a common environment used by  $n$  unevenly sized parties (firms, countries, or regions of countries) indexed by  $i = 1, \dots, n$ , so our theoretical results apply generally to well-mixed pollutants. An absent subscript  $i$  means summation ( $Z := \sum_i Z_i$  for any  $\{Z_i\}$ ); while a tilde,  $\sim$ , means an uncertain (stochastic) variable, and its absence denotes an expectation ( $Z_i = E[\tilde{Z}_i]$ ). Each party's uncertain, BAU emission in say tonnes/year (t/yr) at a single future date is<sup>2</sup>:

$$\tilde{E}_i^b = E_i^b(1 + \varepsilon_{Ei}),$$

where<sup>3</sup>

$$\varepsilon_{Ei} = \text{proportional uncertainty in } i\text{'s emission, with } E[\varepsilon_{Ei}^2] =: \sigma_{Ei}^2 \quad (1)$$

and importantly, all errors are assumed independent with zero means<sup>4</sup>:

$$E[\varepsilon_{Ei}\varepsilon_{Ek}] = 0 \quad \forall i \neq k, \quad \text{and} \quad E[\varepsilon_{Ei}] = 0. \quad (2)$$

Each party abates its emissions by an uncertain  $\tilde{Q}_i$  t/yr in response to a tax or trading mechanism created by an "authority" (a global treaty or a national law, with full participation and enforcement assumed). We will compare the market-wide (i.e., social) net benefits for each mechanism of achieving a given target  $X$  for expected total emissions:

$$X = E[\sum_i (\tilde{E}_i^b - \tilde{Q}_i)] = E^b - Q, \quad (3)$$

given that each party's uncertain abatement cost is  $\tilde{C}_i(\tilde{Q}_i)$  in say \$/yr.<sup>5</sup>

With an emissions tax, denoted  $p^P$  for Price, the authority chooses a certain tax rate  $p^P$  (in \$/t) so that  $Q$ , the expected sum of abatements  $\tilde{Q}_i(p^P)$  – which each party chooses so as to equate its marginal abatement cost (MAC)  $\tilde{C}'_i(\tilde{Q}_i)$  with  $p^P$  – equals the expected abatement task,  $E^b - X$  (assumed positive). The authority also levies only some share  $\rho$  of the potential, expected tax revenue  $p^P X$ , by giving each party  $i$  a tax threshold  $(1 - \rho)X_i$  ( $0 \leq \rho \leq 1$ ,  $X_i > 0$ ) as a quasi-property right.<sup>6</sup> This preserves long-run, partial-equilibrium efficiency by making  $p^P$  apply to emitters' exit-entry decisions, but does not affect  $Q$ . Under certainty a threshold  $(1 - \rho)X_i$  is symmetric with giving  $(1 - \rho)X_i$  free tradable permits (Pezzey [12], who called the tax threshold a "baseline for a charge-subsidy"). It is thus an inframarginal tax exemption for  $i$ , as distinct from exempting  $i$  completely from the tax, here called an exclusion. Exclusions and dilutions (lower rates for selected emitters) are frequent practical occurrences (see for example Svendsen et al. [19]), but they remain outside our model.

<sup>1</sup> The abatement cost term in our tax-vs-trading advantage, derived independently, is in fact a special case of results for imperfectly mixed emissions in Williams [22]; but he did not stress the multi-party question and gave no empirical results. A multi-party tax-vs-trading advantage can be computed from Mandell [8], but only for a specific form of cross-party variation in marginal abatement cost slopes.

<sup>2</sup> In estimating  $E[\varepsilon_{Ei}^2]$  empirically, Jotzo and Pezzey (p263) combined uncertainties in GDP, in emissions intensities of GDP, and in non-GDP-linked emissions.

<sup>3</sup> Note the distinction between  $E[\cdot]$  for the expectation operator, and  $E_i^b$ ,  $E^b$ , etc for various measures of BAU emissions.

<sup>4</sup> The effect of positive cross-party emission correlations is separately addressed in Section 2.2.

<sup>5</sup> We thank a referee for noting that welfare maximisation for a tax and for trading do not necessarily result in the same expected abatement  $Q$ , unless marginal abatement costs and benefits are linear, as in fact we assume below.

<sup>6</sup> The only constraint on the  $\{X_i\}$  here is that  $\sum_i X_i = X$ , so assuming the same  $\rho$  for all parties still allows the authority to choose the distribution  $\{(1 - \rho)X_i\}$  on political grounds.

With emissions trading, denoted  $T$ , the authority creates  $X$  tradable permits (again  $< E^b$ ), auctions  $\rho X$  permits, and gives each party  $(1 - \rho)X_i$  free permits.<sup>7</sup> Permit trading then establishes an uncertain permit price  $\tilde{p}^T$  (unaffected by  $\rho$ ), and each party chooses abatement  $\tilde{Q}_i(\tilde{p}^T)$  so as to equate its MAC  $\tilde{C}'_i(\tilde{Q}_i)$  to this price. Permit-market clearing ensures that total abatement  $\tilde{Q}_i(\tilde{p}^T)$  equals the required abatement  $\tilde{E}^b - X$ .

Denoting either tax rate  $p^P$  or permit price  $\tilde{p}^T$  in our model by emission price  $\tilde{p}$ , with either tax or trading the authority thus gets revenue  $\tilde{R}_i := \tilde{p}[\tilde{E}_i^b - \tilde{Q}_i(\tilde{p}) - (1 - \rho)X_i]$  from party  $i$ . This will be negative for any party  $i$  whose abated emissions fall below its threshold or free permit level (that is, when  $\tilde{E}_i^b - \tilde{Q}_i(\tilde{p}) < (1 - \rho)X_i$ ); but we assume  $\rho$  is chosen high enough to make total revenue  $\tilde{R}$  positive.

We call  $\rho$  the mechanism's *efficient recycling share*, but this term needs careful explanation.  $\rho$  is the share of potential revenue  $\tilde{p}X$  which is both raised (rather than not raised in the first place by giving tax thresholds or free permits) and recycled efficiently (as lower rates of existing factor taxation, thus lowering the welfare cost of existing, distortionary taxation, rather than as lump sums). Efficient recycling is assumed by almost all tax-interaction literature, even though it has rarely happened in practice.<sup>8</sup> We retain this assumption because the welfare benefits of commonly-seen ways of recycling revenue, such as compensation for low-income households or support for low-carbon technologies, are much harder to measure; and it is useful to have both a reference measure of revenue-recycling welfare gain, and comparability with the tax-interaction literature.

The authority induces abatement in order to achieve environmental benefits and thus raise welfare. Given perfect emission mixing, party  $i$ 's benefit depends on total abatement  $\tilde{Q}$ , and is denoted  $\tilde{B}_i(\tilde{Q})$  (\$/yr). We then take the approximate net social benefit attributable to party  $i$  of abatement, compared to zero abatement everywhere, to be

$$\tilde{A}_i(\tilde{Q}, \tilde{Q}_i) := \tilde{B}_i(\tilde{Q}) - \tilde{C}_i(\tilde{Q}_i) - \mu \tilde{C}_i(\tilde{Q}_i) - \mu' \tilde{p}(1 - \rho)X_i, \quad (4)$$

and assume the authority chooses parameters so as to maximise risk-neutral welfare, defined as expected total net benefit  $A (= E[\sum_i \tilde{A}_i])$ . Here  $\mu > 0$  is the marginal cost of public funds minus one caused by distortionary factor taxation, and  $\mu' > \mu$  is the marginal excess burden when tax revenue is returned to households as lump-sum transfers (or equivalently, when it is not raised in the first place by giving out emission tax thresholds or free permits). This gives rise to two approximate general equilibrium social costs:  $\mu \tilde{C}_i(\tilde{Q}_i)$  from emission price  $\tilde{p}$  interactions with the factor tax, and  $\mu' \tilde{p}(1 - \rho)X_i$  from lost revenue-recycling benefit caused by the share  $(1 - \rho)$  of revenue not raised and recycled (by analogy from Goulder et al. [5, pp. 335–342]; see the online appendix accessible from [www.aere.org/journals](http://www.aere.org/journals)). These costs are approximate because the sizes and even signs of  $\mu$  and  $\mu'$  are now so contentious (see Section 3.1), which makes the ideal procedure of constructing and numerically solving a full general equilibrium model of little extra value for practical policymaking. For any greenhouse gas application,  $\mu$  and  $\mu'$  will obviously vary across parties (countries), but to keep analysis tractable we need to assume all  $\mu_i = \mu$  and  $\mu'_i = \mu'$ .

Finally, we assume quadratic cost and benefit functions<sup>9</sup>:

$$\tilde{C}_i(\tilde{Q}_i) := \frac{1}{2}(1/M_i)\tilde{Q}_i^2 + \varepsilon_{Ci}\tilde{Q}_i, \text{ where} \quad (5)$$

$\{M_i\} > 0$  are parameters, and  $1/M$  is the total MAC curve's slope;  
 $\varepsilon_{Ci}$  is  $i$ 's uncertainty in MAC, with<sup>10</sup>

$$E[\varepsilon_{Ci}] = 0 \text{ and } E[\varepsilon_{Ci}^2] = \sigma_{Ci}^2 \text{ for all } i,$$

and each  $\varepsilon_{Ci}$  is assumed independent of all other uncertainties in the model. (6)

$$\tilde{B}_i(\tilde{Q}) := V_i\tilde{Q} - \frac{1}{2}W_i\tilde{Q}^2, \text{ where } V_i > 0, W_i > 0. \quad (7)$$

We call  $V$  the linear valuation of abatement, while  $W$  is the slope of the total marginal abatement benefit (MAB) curve.<sup>11</sup> The net social benefit (4) attributable to party  $i$  is thus

$$\tilde{A}_i = V_i\tilde{Q} - \frac{1}{2}W\tilde{Q}^2 - (1 + \mu)\left[\frac{1}{2}(1/M_i)\tilde{Q}_i^2 + \varepsilon_{Ci}\tilde{Q}_i\right] - \mu'(1 - \rho)X_i\tilde{p}. \quad (8)$$

<sup>7</sup> Jotzo and Pezzey's uncertain targets  $\{\tilde{X}_i\}$  included intensity (indexed) targets, but this extension would distract from our focus here.

<sup>8</sup> A notable exception is the Australian carbon pricing legislation due to come into effect in mid-2012 [2]. This will recycle about half of the permit revenue to households, mainly as reductions in income taxes.

<sup>9</sup> A referee noted that this additive form of uncertainty, used by Weitzman (Eq. (10)) and many authors since, makes  $\tilde{C}'_i(0) = \varepsilon_{Ci}$ , so MAC could be negative at zero abatement. This awkward possibility would be avoided by assuming multiplicative uncertainty, as in Hoel and Karp [6]. We still use (5), both to make our results comparable to the Weitzman-inspired literature, and because we consider only large-abatement situations where  $\text{MAC} < 0$  is very unlikely.

<sup>10</sup> The independence assumption is crucial for our results and is discussed below.

<sup>11</sup> We ignore any shift stochasticity in  $i$ 's MAB, which does not affect the tax-versus-trading comparison. We thus also ignore any correlations between MAB and MAC uncertainties, which do affect the comparison [18]. For nations' emissions of greenhouse gases, there is no evidence for such correlation.

We can then show (see online material) that the optimal welfare from an emissions tax mechanism is

$$A^P = A^* + \frac{1}{2} \Sigma_i [(1+\mu)/M_i - W] M_i^2 \sigma_{Ci}^2, \quad (9)$$

and from an emissions trading mechanism is

$$A^T = A^* + \frac{1}{2} \Sigma_i [(1+\mu)/M_i - (1+\mu)/M] M_i^2 \sigma_{Ci}^2 - \frac{1}{2} [(1+\mu)/M + W] \Sigma_i (E_i^b)^2 \sigma_{Ei}^2 \quad (10)$$

where welfare under certainty from either mechanism is

$$A^* := \frac{1}{2} [V - (1-\rho)\mu' E^b / M]^2 M / [1 + \mu + WM - 2(1-\rho)\mu']. \quad (11)$$

Hence the *tax-versus-trading* (welfare) *advantage* is

$$\Delta := A^P - A^T = \frac{1}{2} [(1+\mu)/M - W] \Sigma_i M_i^2 \sigma_{Ci}^2 + \frac{1}{2} [(1+\mu)/M + W] \Sigma_i (E_i^b)^2 \sigma_{Ei}^2; \quad (12)$$

while the optimal, expected emission price and total abatement are

$$p^* = [V - (1-\rho)\mu' E^b / M] / [1 + \mu + WM - 2(1-\rho)\mu'], \quad Q^* = Mp^*. \quad (13)$$

For use later, we also write the trading welfare including only abatement-cost uncertainties ((10) minus the last term) as

$$A_C^T := A^* + \frac{1}{2} \Sigma_i [(1+\mu)/M_i - (1+\mu)/M] M_i^2 \sigma_{Ci}^2. \quad (14)$$

An implicit assumption here is that price  $p^* > 0$ , hence the efficient recycling share  $\rho > 1 - (VM/\mu' E^b)$  when  $\mu' > 0$ ; otherwise marginal abatement actually lowers welfare, as stressed by Goulder et al. [5]. For values of  $\rho$  and  $\mu'$  plausible in our greenhouse application, this is a feasible, though far from non-trivial condition, as will be seen.

## 2.2. Features and qualifications of the total expected welfare results

Three features of the above results deserve comment. The first, novel and empirically most important economic result is that in (12), BAU emissions uncertainties add  $\frac{1}{2} [(1+\mu)/M + W] \Sigma_i (E_i^b)^2 \sigma_{Ei}^2$  to the tax-versus-trading advantage. Intuitively, this arises from emissions uncertainties being a second source of emission price uncertainty under trading, and thus a second advantage of the fixed price under a tax. It deserves a good deal more attention, because in our global, greenhouse case study, it easily dominates the first, abatement-cost term in (12).

Next, note that in our model the efficient recycling share  $\rho$  affects only the certainty welfare  $A^*$ , not the advantage  $\Delta$ . So in principle the choice of efficient recycling share is separate from the tax-versus-trading choice. In practice, though, we think current institutional and political realities mean the two choices are tightly connected and for that reason alone need to be considered together, as opined in our Conclusions.

A third, more complex feature, though empirically less important for our greenhouse case, is the effect of including multiple, independent parties on the first, abatement cost term in the advantage (12), which we write as<sup>12</sup>

$$\Delta_C := \frac{1}{2} [(1+\mu)/M - W] \Sigma_i M_i^2 \sigma_{Ci}^2 \quad (15)$$

This is indeed an advantage ( $\Delta_C > 0$ ) for short-run greenhouse emissions, because the large existing pollutant stock means the total MAB curve is relatively very flat ( $W \ll (1+\mu)/M$ ) [15]. But if a single representative party, denoted 1, is assumed, (15) becomes

$$\Delta_{C1} := \frac{1}{2} [(1+\mu)/M - W] M^2 \sigma_C^2 \quad (16)$$

After converting notation and setting  $\mu=0$ ,  $\Delta_{C1}$  is the main result given by Weitzman's [20] result (20).<sup>13</sup> The ratio  $\Delta_{C1}/\Delta_C$  depends on the distribution of party sizes, but is probably much larger than one, and is about 9 in our empirical model. So the single-party formula  $\Delta_{C1}$ , which ignores how trading dampens the transmission of many parties' (independent) cost uncertainties into uncertainty in the permit price, significantly overestimates the advantage of a tax over realistic permit trading (or of trading over tax, in a different empirical case with  $W > (1+\mu)/M$ ). This could matter, because the well-known, climate-related literature on prices-versus-quantities uses the single-party formula  $\Delta_{C1}$  (with  $\mu=0$ ): see for example Hoel and Karp [6], Pizer [15] and Newell and Pizer [10]. We could not find any empirical study of tax-versus-trading for greenhouse emissions abatement which both uses Weitzman's theoretical foundation and allows for multi-party trading.

However, the tax-versus-trading advantage is partly restored in the greenhouse case by two considerations about correlations, which remain off-model in the absence of suitable data.<sup>14</sup> First, while some determinants of abatement costs like fuel mix vary greatly across countries, other determinants like energy-saving technologies and before-tax fuel prices are now fairly globalised. So in practice there will be some positive, cross-party correlation in cost uncertainties [17], and

<sup>12</sup> With converted notation,  $\mu=0$  and changed sign, this is Williams' [22] result (34) for the case "when the goods are perfect substitutes", as with globally well-mixed pollution.

<sup>13</sup> Though Weitzman's footnote 1 on p490 clearly envisaged an application to emissions trading, he did not give any multi-party trading formula. His Section 5 computed the many-party advantage of prices over *non-traded* quantities.

<sup>14</sup> We thank the referees for both these points.

**Table 1**  
Key global parameters in 2020 in MATES.

Parameter	Notation and value
BAU greenhouse gas emissions	$E^b = 54.4 \text{ Gt/yr } (= 1.3E_{2002})$
Linear valuation of abatement	$V = 21.9 \text{ (\$/t)}$
MAC slope	$1/M = 2.32 \text{ (\$/t)/(Gt/yr)}$
MAB slope	$W = 0.22 \text{ (\$/t)/(Gt/yr)}$
Uncertainties in abatement costs	$\sum_i M_i \sigma_{Ci}^2 = 7.60 \text{ (G\$/yr)}$
	$\sum_i M_i^2 \sigma_{Ci}^2 = 0.37 \text{ (Gt/yr)}^2$
Uncertainties in BAU emissions	$\sum_i (E_i^b)^2 \sigma_{Ei}^2 = 11.35 \text{ (Gt/yr)}^2$

\$=US\$ in 2000; t=tonne of CO<sub>2</sub>-equivalent.

our independence assumption in (6) is overstated. Indeed, under the opposite, also overstated, assumption of identical, perfectly correlated  $\{\varepsilon_{Ci}\}$ ,  $\Delta_C$  reverts to the single-party form  $\Delta_{C1}$  (see online material). Second, if emissions uncertainties are correlated rather than independent across parties, so that  $E[\varepsilon_{Ei}\varepsilon_{Ek}] =: \sigma_{Eik} \neq 0$  for several  $i \neq k$ , then  $\sum_i (E_i^b)^2 \sigma_{Ei}^2$  in (10) and (12) is replaced by  $\sum_i (E_i^b)^2 \sigma_{Ei}^2 + 2 \sum_{i \neq k} E_i^b E_k^b \sigma_{Eik}$  (see online material). Assuming positive correlations outweigh negative ones ( $\sum_{i \neq k} E_i^b E_k^b \sigma_{Eik} > 0$ ), as seems reasonable for example in the 2008 global recession, this also increases the tax-versus-trading advantage.

### 3. Empirical results for climate policy

#### 3.1. Parameters chosen

Our application of MATES is to static abatement of greenhouse emissions in 2020, in a world with 18 regions that differ greatly in GDP and other parameters. The parameters used to calculate results (9)–(16) for optimal, global welfare are in Table 1; their empirical calibration is explained in Jotzo and Pezzey [7].

The remaining numbers used in our model are  $\rho$ , the efficient recycling share;  $\mu$ , the marginal cost of public funds minus one; and  $\mu'$ , the marginal excess burden. The efficient recycling share  $\rho$  is chosen by national governments under strong political constraints. Most economists assume  $\mu' > 0$ , hence a potential revenue-recycling efficiency gain, and thus recommend full revenue raising and recycling ( $\rho=1$ ) on welfare grounds. But with emissions trading, governments typically have found it difficult to resist industry lobbying to give away many tradable permits for free, at least initially. So as useful values to consider, without prejudging which may be most realistic, we choose

$$\rho = 1 \text{ or } 0.5 \text{ for all regions.} \quad (17)$$

For our main results we choose the benchmark values from Goulder et al. [5, p335 and p342],

$$\mu = 0.1 \text{ and } \mu' = 0.3. \quad (18)$$

However, estimates of  $\mu$  and  $\mu'$  are now very contentious. By including the status (relative consumption) externalities, Wendner and Goulder [21] estimated a range of  $\mu'$  from  $-0.27$  to  $0.91$ , depending on other parameters chosen, significantly lower than previous authors' estimates. So in the next section we also discuss the case  $\mu'=0$ , which greatly changes one of our results.

#### 3.2. Results

Table 2 shows results, from inserting values from Table 1 and (17)–(18) into (9)–(16), in four sections. The first gives total welfare from a tax ( $A^P$  in (9)) and from trading ( $A^T$  in (10)), and trading welfare with abatement-cost uncertainties only ( $A_C^T$  in (14)). The second shows that our complete, multi-party, tax-versus-trading advantage ( $\Delta$  in (12)) greatly exceeds the advantage with only abatement-cost uncertainties counted ( $\Delta_C$  in (15)). (No different  $\rho$  or  $V$  values are shown, since neither parameter affects the  $\Delta$  formulae.) This reflects how emissions uncertainties dominate the  $\sum_i M_i^2 \sigma_{Ci}^2$  term for abatement-cost uncertainties in Table 1. As discussed in Section 2.2 and shown here, this dominance would be weakened only slightly if one assumed perfect correlation in abatement cost uncertainties and thus replaced  $\Delta_C$  with the single-party result  $\Delta_{C1}$ , about nine times larger than  $\Delta_C$ . As also discussed there, dominance would be strengthened by likely correlations in emissions uncertainties.

Yet Table 2's third section highlights a striking feature in the first section, that the full tax-versus-trading advantage of 16.2 G\$/yr is in turn dominated by the welfare loss of 84.3 G\$/yr caused by lowering the efficient revenue-recycling share  $\rho$  from 1 to 0.5. This loss means emission pricing has negligible or even negative welfare when  $\rho=0.5$ ; and the loss more than doubles again under the sensitivity test of doubling the linear valuation  $V$ . However, all this crucially depends on the benchmark value of  $\mu'=0.3$ . If the marginal excess burden instead happens to be  $\mu'=0$  for status reasons discussed after (18), then from (11), varying  $\rho$ , the extent of revenue raising and recycling, has absolutely no effect on welfare.



**Table 2**

MATES results for global greenhouse abatement in 2020.

Marginal cost of public funds – 1 ( $\mu$ )	0.1	0.1
Marginal excess burden ( $\mu'$ )	0.3	0.3
Efficient recycling share of global target, $\rho$ ( $=1 - \text{tax thresholds' or free permits' share}$ )	1 (“pure” mechanism)	0.5
Welfare in 2020 (expected global net benefit in US2000 G\$/yr) from:		
Optimal emissions tax with thresholds ( $A^P$ )	90.6	6.3
Optimal emissions trading with abatement-cost and emissions uncertainties ( $A^T$ )	74.4	–9.9
Optimal emissions trading with abatement-cost uncertainties only ( $A_C^T$ )	90.2	5.8
Optimal tax-versus-trading welfare advantage (independent of $\rho$ or $V$ ) from:		
Abatement cost and emissions uncertainties, multi-party ( $\Delta = A^P - A^T$ )	16.2	16.2
Abatement-cost uncertainties only, multi-party ( $\Delta_C = A^P - A_C^T$ )	0.4	0.4
Abatement-cost uncertainties only, single-party ( $\Delta_{C1}$ )	3.8	3.8
Welfare lost from only 0.5 efficient recycling share ( $A^*(\rho=0.5) - A^*(\rho=1)$ ), for either optimal mechanism:		
With standard linear valuation $V$	–	–84.3
With doubled linear valuation	–	–196.9
Other expected results for either optimal mechanism:		
Emission price $p^*$ (\$/t CO <sub>2</sub> -equivalent)	18.3	3.3
Abated emissions $E^b - Q^*$ , cf. BAU emissions in 2020	–15%	–3%

Lastly, the higher expected global emission price  $p^* = 18.3$  \$/t CO<sub>2</sub> from (13), and corresponding 15% abatement, are modest compared to current climate policy aims [1], though broadly in line with forward prices in the EU permit market. And the abatement cost associated with  $Q^*$  (not shown) is less than 0.1% of MATES' projected global GDP in 2020, confirming this is an acceptable application of our mainly partial equilibrium model.

A robust, overall effect of our analysis is thus to sharply increase the estimated tax-versus-trading welfare advantage. More complex but no less important is our result that, depending solely on the value of the marginal excess burden, such advantage might either be lost many times over, or fairly unaffected, if the efficient recycling share used with both mechanisms is well below 1. This ambiguous sensitivity suggests both that tax-versus-trading and efficient recycling shares are issues well worth considering simultaneously, and that **better, country-specific estimates of the marginal excess burden are badly needed.**

#### 4. Conclusions

We have presented the first multi-party, theoretical and empirical model of greenhouse gas abatement that combines the issues of tax-versus-trading under uncertainty, and revenue recycling. Our mainly partial-equilibrium model has the added novelty of including uncertainties in business-as-usual emissions as well as in marginal abatement costs. Theoretically, we showed that if parties' abatement-cost uncertainties are independent, then emissions trading dampens shocks in abatement costs, so the welfare advantage of a tax over trading from this source is much lower than the single-party result found by Weitzman in 1974 and used by almost all literature since. But empirically and more importantly, we also showed that the global lowering of this advantage is overwhelmed by the much larger tax-versus-trading advantage from a tax's better handling of emissions uncertainties. So overall our results substantially boost the welfare case for using a carbon tax instead of trading, on top of arguments advanced by authors like Nordhaus [11].

However, a yet much larger, but more contentious welfare advantage may come from raising a mechanism's “efficient recycling share”, by giving fewer tax thresholds or free tradable permits as a share of abated emissions, and thus raising and recycling more revenue. The “may” is needed both because, even under the standard assumption that all such revenue is recycled efficiently as factor tax cuts, the welfare benefit of doing so is now contentious, and could even be negligible; and because carbon pricing revenues have rarely been spent on factor tax cuts in practice. Despite this imprecision, the great potential welfare benefit from revenue recycling suggests that preferring a tax to trading, and raising the efficient recycling share, are issues well worth considering together.

Such consideration seems bound to pit economics against politics and practicality. **From over twenty years of evidence, we contend that trying to use pure emissions taxation at an optimal rate (and thus maximise welfare in Table 2) will fail, and could even be counterproductive in the short term. This is because of the political unacceptability of full revenue raising, and the institutional unavailability of the emissions tax thresholds that would allow partial revenue raising to remain efficient under a tax [13,14].**

If this opinion is accepted, then research would be worthwhile on the political economy of finding the best feasible alternative to pure, optimal taxation in various circumstances. When and where might that be an initially low but rising tax rate, or using tax thresholds as quasi-property rights, or a tax with significant exclusions and dilutions, or emissions trading with some free permits? Our analysis also shows the need for more economic research on four topics. First, on a general-equilibrium model with both tax-versus-trading under uncertainty, and

revenue-recycling, which would fill an important gap in the theoretical literature. Second, on the correlations among both emissions uncertainties and abatement-cost uncertainties. Third, on estimating the marginal excess burden. Fourth, on the welfare benefits of the ways carbon pricing revenues are likely to be spent in practice, such as compensation for low-income households or support for low-carbon technologies.

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## Appendix A. Further derivations

Further derivations associated with this article can be found in the online appendix, which can be accessed from [www.aere.org/journals](http://www.aere.org/journals).

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### Supplementary Material

to "Tax-versus-trading and efficient revenue recycling as issues for greenhouse gas control"  
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#### I. DERIVATION OF COST TERMS IN (4)

The unpublished Appendix A to Goulder et al. [5] gives (in notation used in the published paper) the following linearly approximated formulae. Primary cost  $\Delta W^A + \Delta W^O$ , revenue-recycling benefit  $\Delta W^R$  and tax-interaction cost  $\Delta W^I$ , are respectively, in terms of emission price  $t_E$ , marginal cost of public funds minus one  $M$ , initial emissions  $E_0$  and abatement  $\Delta E$  of a single representative emitter:

$$(A.14) \quad \Delta W^A + \Delta W^O = t_E \Delta E / 2; \quad \Delta W^R = M t_E (E_0 - \Delta E); \quad \Delta W^I = M t_E (E_0 - \Delta E) / 2$$

Of these,  $\Delta W^R$  must be amended to allow for the partial revenue-recycling which occurs in our paper because of intermediate levels of "free emissions" (tax thresholds or free permits). With the efficient recycling share  $\rho = 1$ , the revenue-recycling benefit  $\Delta W^R$  above is the definite integral of  $\partial W^R = [(d/dt_E)(M t_E E)]$ , a term in Goulder et al.'s (2.10), from emission price 0 to price  $t_E$ , assuming emissions  $E$  decline linearly from  $E_0$  to  $E_0 - \Delta E$  as this happens. However, when  $\rho < 1$ , a proportion  $(1-\rho)$  of abated emissions  $E_0 - \Delta E$  is "free", resulting in some revenue not being raised and recycled, but remaining with households. This causes an income effect on labour supply, which raises the marginal welfare cost per dollar of tax revenue *not* raised from  $M$  to  $M'$ , the marginal excess burden. The partial revenue-recycling derivative then changes to  $\partial W^{Rp} = [(d/dt_E)[M t_E E - M' t_E (1-\rho)(E_0 - \Delta E)]]$ , which when integrated between 0 and  $t_E$  gives  $M t_E (E_0 - \Delta E) - M' t_E (1-\rho)(E_0 - \Delta E) =: \Delta W^{Rp}$ .

The linearised general equilibrium welfare cost of emission pricing with efficient recycling share  $\rho$  is then the primary cost minus the partial recycling benefit plus the tax-interaction cost:



$$\begin{aligned}
& \Delta W^A + \Delta W^O - \Delta W^{Rp} + \Delta W^I \\
&= t_E [\Delta E/2 - M(E_0 - \Delta E) + M'(1-\rho)(E_0 - \Delta E) + M(E_0 - \Delta E/2)] \\
&= (1+M)(t_E \Delta E/2) + M' t_E (1-\rho)(E_0 - \Delta E).
\end{aligned} \tag{A1.1}$$

Dividing by  $\Delta W^A + \Delta W^O = t_E \Delta E/2$  and setting  $\rho = 1$  or  $0$  then respectively gives Goulder et al.'s results (2.11) for a pure tax, or (2.11a) for tradable, completely non-auctioned permits.

Our mainly partial-equilibrium model does not have Goulder et al.'s general-equilibrium foundation of a utility function and government budget constraint; but as stated in the main paper, given the contention over the sizes of  $M$  and  $M'$ , the extra accuracy of a general equilibrium model is of little extra value in our context. Our model has direct equivalents, allowing for our inclusion of our abatement cost uncertainties with no underlying general-equilibrium foundation, for the above primary cost ( $\tilde{C}_i$  in place of  $t_E \Delta E/2$  above, which entails no further approximation since our marginal cost from (5) is already linear); for expected abated emissions ( $E_i^b - Q_i$  in place of  $E_0 - \Delta E$ ; and for a single party  $(1-\rho)(E_i^b - Q_i) = (1-\rho)X_i$ , the amount of tax thresholds or free permits granted). Making the remaining conversions to our notation ( $M \rightarrow \mu$  for the marginal cost of public funds minus one,  $M' \rightarrow \mu'$  for the marginal excess burden, and  $t_E \rightarrow \tilde{p}$  for the emission price) then converts (A1.1) to  $(1+\mu)\tilde{C}_i + \mu'\tilde{p}(1-\rho)X_i$ , the social cost of abatement deducted on (4)'s right-hand side.

## II. MODEL RESULTS (ALL UNCERTAINTIES INDEPENDENT)

For convenience, we repeat the social net benefit for party  $i$ :

$$\tilde{A}_i = V_i \tilde{Q} - \frac{1}{2} W_i \tilde{Q}^2 - (1+\mu)[\frac{1}{2}(1/M_i) \tilde{Q}_i^2 + \epsilon_{Ci} \tilde{Q}_i] - \mu'(1-\rho)X_i \tilde{p} \tag{8}$$

For a *tax with thresholds*, where  $\tilde{p} = p^P$ , we combine the price-equals-MAC rule  $\tilde{C}_i'(\tilde{Q}_i) = p^P$  and the quadratic total cost function in (5) to give

$$\begin{aligned}
& (1/M_i) \tilde{Q}_i + \epsilon_{Ci} = p^P \\
\Rightarrow \tilde{Q}_i &= M_i p^P - M_i \epsilon_{Ci}, \quad \tilde{Q} = M p^P - \Sigma_i M_i \epsilon_{Ci}, \quad Q = M p^P
\end{aligned} \tag{A1.2}$$

$$\begin{aligned}
\text{so } \tilde{Q}_i^2 &= (M_i p^P)^2 + (M_i \epsilon_{Ci})^2 - 2M_i^2 p^P \epsilon_{Ci} \\
\Rightarrow E[\tilde{Q}_i^2] &= M_i^2 [(p^P)^2 + \sigma_{Ci}^2]
\end{aligned} \tag{A1.3}$$

$$\begin{aligned} \text{also } \tilde{Q}^2 &= (Mp^P)^2 - 2Mp^P \sum_i M_i \varepsilon_{Ci} + (\sum_i M_i \varepsilon_{Ci})^2 \\ \Rightarrow E[\tilde{Q}^2] &= (Mp^P)^2 + \sum_i M_i^2 \sigma_{Ci}^2 \quad (\text{using independence in (6)}) \end{aligned} \quad (\text{A1.4})$$

$$\begin{aligned} \text{also } \varepsilon_{Ci} \tilde{Q}_i &= \varepsilon_{Ci} M_i p^P - M_i \varepsilon_{Ci}^2 \\ \Rightarrow E[\varepsilon_{Ci} \tilde{Q}_i] &= -M_i \sigma_{Ci}^2 \end{aligned} \quad (\text{A1.5})$$

So taking expectations of (8) and using (A1.2)-(A1.5) gives

$$\begin{aligned} A_i^P &= V_i M p^P - \frac{1}{2} W_i [(M p^P)^2 + \sum_i M_i^2 \sigma_{Ci}^2] \\ &\quad - (1+\mu) \{ [\frac{1}{2} M_i [(p^P)^2 + \sigma_{Ci}^2] - M_i \sigma_{Ci}^2] - \mu'(1-\rho) X_i p^P \}; \end{aligned} \quad (\text{A1.6})$$

and summing gives expected total net benefit for using the tax:

$$\begin{aligned} A^P &= V M p^P - \frac{1}{2} W [(M p^P)^2 + \sum_i M_i^2 \sigma_{Ci}^2] \\ &\quad - \frac{1}{2} (1+\mu) [M (p^P)^2 - \sum_i M_i \sigma_{Ci}^2] - \mu'(1-\rho) X p^P, \end{aligned}$$

which using  $X = E^b - M p^P$  (from (3) and (A1.2)) means

$$A^P = \bar{A}(p^P) + \frac{1}{2} \sum_i [(1+\mu)(1/M_i) - W] M_i^2 \sigma_{Ci}^2, \quad \text{where} \quad (\text{A1.7})$$

$$\bar{A}(p^P) := V M p^P - \frac{1}{2} M (1+\mu + W M) (p^P)^2 - (1-\rho) \mu' (E^b - M p^P) p^P. \quad (\text{A1.8})$$

The optimal tax rate is found by setting  $\partial A^P / \partial p^P = 0$ :

$$\begin{aligned} \partial A^P / \partial p^P &= V M - M (1+\mu + W M) p^P - \mu'(1-\rho) E^b + 2(1-\rho) \mu' M p^P \\ &= [V - \mu'(1-\rho) E^b / M] M - [1+\mu + W M - 2(1-\rho) \mu'] M p^P = 0; \end{aligned}$$

so optimally,

$$p^P = [V - \mu'(1-\rho) E^b / M] / [1+\mu + W M - 2(1-\rho) \mu'] =: p^* \quad \text{as in (13)}$$

(which is valid only if  $\rho > 1 - VM/\mu' E^b$ , as discussed after (14)); and

$$\begin{aligned} \bar{A}(p^*) &= [V - (1-\rho) \mu' E^b / M] M p^* - \frac{1}{2} M [1+\mu + W M - 2(1-\rho) \mu'] (p^*)^2 \\ &= \frac{1}{2} [V - (1-\rho) \mu' E^b / M]^2 M / [1+\mu + W M - 2(1-\rho) \mu'] =: A^* \end{aligned} \quad \text{as in (11);}$$

$$\text{so } A^P = A^* + \frac{1}{2} \sum_i [(1+\mu)(1/M_i) - W] M_i^2 \sigma_{Ci}^2 \quad \text{as in (9).}$$

For *tradable permits*, we have  $\tilde{p} = \tilde{p}^T$ , and  $\tilde{C}_i'(\tilde{Q}_i) = \tilde{p}^T$  and (5) give

$$(1/M_i)\tilde{Q}_i + \varepsilon_{Ci} = \tilde{p}^T$$

$$\Rightarrow \tilde{Q}_i = M_i\tilde{p}^T - M_i\varepsilon_{Ci}, \quad \tilde{Q} = M\tilde{p}^T - \Sigma_i M_i\varepsilon_{Ci}, \quad \text{and} \quad Q = Mp^T. \quad (\text{A1.9})$$

From (1) and (A1.9), total abatement  $\tilde{Q}(\tilde{p}^T) = \tilde{E}^b - X$  is also:

$$\tilde{Q} = E^b - X + \tilde{E}^b - E^b = Mp^T + \Sigma_i E_i^b \varepsilon_{Ei}, \quad (\text{A1.10})$$

so using the variance, mean and independence assumptions in (1) and (2),

$$\Rightarrow E[\tilde{Q}^2] = (Mp^T)^2 + \Sigma_i (E_i^b)^2 \sigma_{Ei}^2, \quad (\text{A1.11})$$

and (A1.9) and (A1.10) together give

$$M\tilde{p}^T - \Sigma_i M_i \varepsilon_{Ci} = Mp^T + \Sigma_i E_i^b \varepsilon_{Ei}$$

$$\Rightarrow \tilde{p}^T = p^T + (1/M)\Sigma_k (M_k \varepsilon_{Ck} + E_k^b \varepsilon_{Ek})$$

$$\Rightarrow \tilde{Q}_i = M_i[p^T + (1/M)\Sigma_k (M_k \varepsilon_{Ck} + E_k^b \varepsilon_{Ek}) - \varepsilon_{Ci}] \quad (\text{A1.12})$$

$$\Rightarrow \varepsilon_{Ci}\tilde{Q}_i = M_i[p^T + (1/M)\Sigma_k (M_k \varepsilon_{Ck} + E_k^b \varepsilon_{Ek}) - \varepsilon_{Ci}] \varepsilon_{Ci}$$

$$\Rightarrow E[\varepsilon_{Ci}\tilde{Q}_i] = [(1/M)M_i^2 - M_i] \sigma_{Ci}^2 = (1/M - 1/M_i) M_i^2 \sigma_{Ci}^2. \quad (\text{A1.13})$$

Also, from (A1.12),

$$\tilde{Q}_i^2 = M_i^2[p^T + (1/M)\Sigma_k (M_k \varepsilon_{Ck} + E_k^b \varepsilon_{Ek}) - \varepsilon_{Ci}]^2$$

$$= M_i^2 [(p^T)^2 + (1/M)^2 [\Sigma_k (M_k \varepsilon_{Ck} + E_k^b \varepsilon_{Ek})]^2 + \varepsilon_{Ci}^2$$

$$+ 2p^T(1/M)\Sigma_k (M_k \varepsilon_{Ck} + E_k^b \varepsilon_{Ek}) - 2p^T \varepsilon_{Ci} - (2/M)\Sigma_k (M_k \varepsilon_{Ck} + E_k^b \varepsilon_{Ek}) \varepsilon_{Ci}]$$

$$\Rightarrow E[\tilde{Q}_i^2] = M_i^2 [(p^T)^2 + (1/M)^2 [\Sigma_k M_k^2 \sigma_{Ck}^2 + (E_k^b)^2 \sigma_{Ek}^2] + \sigma_{Ci}^2 - (2/M)M_i \sigma_{Ci}^2] \quad (\text{A1.14})$$

So taking  $E[\tilde{A}_i^T]$  from (8), using (A1.11)-(A1.14) and  $E[(\Sigma_i E_i^b \varepsilon_{Ei})^2] = (E_i^b)^2 \sigma_{Ei}^2$  from (1) and (2), gives

$$A_i^T = V_i Mp^T - \frac{1}{2} W_i [(Mp^T)^2 + \Sigma_i (E_i^b)^2 \sigma_{Ei}^2]$$

$$- (1+\mu)\frac{1}{2} M_i [(p^T)^2 + (1/M)^2 (\Sigma_k M_k^2 \sigma_{Ck}^2 + \Sigma_i (E_i^b)^2 \sigma_{Ei}^2) + \sigma_{Ci}^2 - (2/M)M_i \sigma_{Ci}^2]$$

$$- (1+\mu)(1/M - 1/M_i)M_i^2 \sigma_{Ci}^2 - (1-\rho)\mu' X_i p^T. \quad (\text{A1.15})$$

Summing then gives

$$\begin{aligned}
A^T &= VMp^T - \frac{1}{2}W[(Mp^T)^2 + \sum_i (E_i^b)^2 \sigma_{Ei}^2] \\
&\quad - \frac{1}{2}(1+\mu)M [(p^T)^2 + (1/M)^2(\sum_k M_k^2 \sigma_{Ck}^2 + \sum_i (E_i^b)^2 \sigma_{Ei}^2)] \\
&\quad - \frac{1}{2}(1+\mu)\sum_i M_i \sigma_{Ci}^2 (1-2M_i/M) - (1+\mu)\sum_i (1/M - 1/M_i)M_i^2 \sigma_{Ci}^2 - (1-\rho)\mu'Xp^T,
\end{aligned}$$

which with  $X = E^b - Mp^T$  from (3) and (A1.9) becomes

$$\begin{aligned}
&= VMp^T - \frac{1}{2}M(1+\mu+WM)(p^T)^2 - (1-\rho)\mu'(E^b-Mp^T)p^T \\
&\quad - \frac{1}{2}[(1+\mu)/M + W]\sum_i (E_i^b)^2 \sigma_{Ei}^2 - (1+\mu)(\frac{1}{2}/M)\sum_i M_i^2 \sigma_{Ci}^2 \\
&\quad - (1+\mu)\sum_i (\frac{1}{2}/M_i - 1/M)M_i^2 \sigma_{Ci}^2 - (1+\mu)\sum_i (1/M - 1/M_i)M_i^2 \sigma_{Ci}^2
\end{aligned} \tag{A1.16}$$

So using  $\bar{A}(\cdot)$  as in (A1.8),

$$A^T = \bar{A}(p^T) + \frac{1}{2}\sum_i [(1+\mu)/M_i - (1+\mu)/M]M_i^2 \sigma_{Ci}^2 - \frac{1}{2}[(1+\mu)/M+W]\sum_i (E_i^b)^2 \sigma_{Ei}^2,$$

where  $\bar{A}(\cdot)$  is as in (A1.8). The same optimisation then applies, giving

$$p^T = p^* \text{ as in (13) and } \bar{A}(p^T) = A^* \text{ as in (11).}$$

Hence

$$A^T = A^* + \frac{1}{2}\sum_i [(1+\mu)/M_i - (1+\mu)/M]M_i^2 \sigma_{Ci}^2 - \frac{1}{2}[(1+\mu)/M+W]\sum_i (E_i^b)^2 \sigma_{Ei}^2 \quad \text{as in (10).}$$

### III. WITH IDENTICAL, PERFECTLY CORRELATED ABATEMENT COST UNCERTAINTIES

If all the  $\{\epsilon_{Ci}\}$  shift uncertainties in MACs are identical and perfectly correlated, we have  $E[\epsilon_{Ci}^2] = E[\epsilon_{Ci}\epsilon_{Ck}] = \sigma_C^2$  for all  $i$  and  $k$ .

For a tax, this changes  $\sum_i M_i^2 \sigma_{Ci}^2$  terms to  $M^2 \sigma_C^2$  in (A1.4) and the  $A^P$  expression before (A1.7); while in that expression,  $\sum_i M_i \sigma_{Ci}^2$  becomes  $M \sigma_C^2$ . (A1.7) is then  $A^P = \bar{A}(p^P) + \frac{1}{2}[(1+\mu)(1/M)-W]M^2 \sigma_C^2$ , and (9) is

$$A^P = A^* + \frac{1}{2}[(1+\mu)(1/M)-W]M^2 \sigma_C^2.$$

For tradable permits,  $E[\varepsilon_{Ci}\tilde{Q}_i]$  in (A1.13) becomes zero. In (A1.14)  $\sum_k M_k^2 \sigma_{Ck}^2$  becomes  $M^2 \sigma_C^2$  while  $(2/M)M_i \sigma_{Ci}^2$  becomes  $2\sigma_C^2$ . The same two changes happen in (A1.15), while the term in  $(1/M - 1/M_i)$  disappears. The last three terms in (A1.16) then become  $-(1+\mu)^{1/2} M \sigma_C^2 + (1+\mu)^{1/2} M \sigma_C^2 - 0$ , so the abatement cost term disappears from  $A^T$ , which becomes just

$$A^T = A^* - \frac{1}{2}[(1+\mu)(1/M) + W] \sum_i (E_i^b)^2 \sigma_{Ei}^2.$$

The first term of the tax-versus-trading advantage  $A^P - A^T$  is thus reduced to  $\frac{1}{2}[(1+\mu)(1/M) - W] M^2 \sigma_C^2$ , which is the single-party result  $\Delta_{C1}$  in (16).

#### IV. WITH CORRELATIONS IN EMISSION UNCERTAINTIES

If  $E[\varepsilon_{Ei} \varepsilon_{Ek}] = \sigma_{Eik} \neq 0$  instead of  $= 0$ , then

$$E[(\sum_i E_i^b \varepsilon_{Ei})^2] = \sum_i (E_i^b)^2 \sigma_{Ei}^2 + 2 \sum_{i \neq k} E_i^b E_k^b \sigma_{Eik},$$

instead of just  $\sum_i (E_i^b)^2 \sigma_{Ei}^2$  as in (A1.11). So in the working from (A1.14) onwards, and hence in the final result for the trading welfare  $A^T$ , all  $\sum_i (E_i^b)^2 \sigma_{Ei}^2$  terms are replaced by  $\sum_i (E_i^b)^2 \sigma_{Ei}^2 + 2 \sum_{i \neq k} E_i^b E_k^b \sigma_{Eik}$ , as stated at the end of Section 2. (We still assume all  $E[\varepsilon_{Ci} \varepsilon_{Ek}] = 0$ , so that  $E[\varepsilon_{Ci} \tilde{Q}_i]$  in (A1.13) is unchanged.)

By contrast,  $\varepsilon_{Ei}$  makes no appearance in  $\tilde{Q}_i$  for a tax, so  $\sigma_{Eik}$  is absent from the tax welfare  $A^P$ .

-oOo-